NEW SERIES, NO. 134 (VOL. XVII) JUNE, 1921

QUARTERLY PUBLICATION OF THE

AMERICAN STATISTICAL ASSOCIATION

Price $1.50 1921 $5.00 Per Annum

Published quarterly by the American Statistical Association
RUMFORD PRESS BUILDING, CONCORD, N. H.
Editorial Office, New York City
Entered at the post-office, Concord, N. H., as second-class mail matter

THE RUMFORD PRESS
CONCORD, N. H.

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THE COEFFICIENT OF CORRELATION

By Franz Boas, *Columbia University.*

Students who are using the coefficient of correlation are aware that the value of the coefficient does not always express the biological, psychological, social, or economic relations which are the subjects of our studies; they realize that its value depends upon a good many extraneous conditions. Since, however, in literature the coefficient is too often used in a mechanical way, and the results of study are generally interpreted as having an immediate bearing upon the problems at issue, without a detailed study of the composition of the series, it seems desirable to call attention to some of the conditions influencing its value.

When two variables are interdependent so that a variant of one partly determines the correlated array of the other, we express their relation by the coefficient of correlation. The derivation of this value is based on the assumption that the averages of arrays are proportional to the determining variant, each being measured from its own average, and on the further assumption that all arrays have equal variability. The former assumption is expressed by

(1a) $[y] = q_1 x$, (brackets indicating averages)

and conversely by

(1b) $[x] = q_2 y$.

From these we derive by multiplication

(2a) $[xy] = q_1[x^2] = q_1\sigma_x^2$

and

(2b) $[xy] = q_2[y^2] = q_2\sigma_y^2$.

It follows that

$$q_1\frac{\sigma_x}{\sigma_y} = q_2\frac{\sigma_y}{\sigma_x}$$

and we call this value r, the coefficient of correlation. Hence

$$r = \frac{[xy]}{\sigma_x \sigma_y}. \tag{3}$$

If we call the variability of an array of y around its own average s, then the mean square variation of the array around the average of the total series to which the array belongs will be

$$s_y^2 + q_1 x^2.$$

The total mean square variability of y, which is σ_y^2, must therefore equal

$$\sigma_y^2 = s_y^2 + q_1^2 \sigma_x^2 \tag{4a}$$

$$\sigma_y^2 = s_y^2 + r^2 \sigma_y^2, \text{ or} \tag{4b}$$

$$s_y^2 = \sigma_y^2(1 - r^2). \quad \text{In the same way} \tag{4c}$$

$$s_x^2 = \sigma_x^2(1 - r^2). \tag{4d}$$

In the series of these values, we are operating therefore with the following seven quantities, σ_x, σ_y, s_x, s_y, q_1, q_2, r.

There are certain relations between these quantities.

$$\left\{ \begin{aligned} q_1 q_2 &= r^2 \\ s_x^2 &= \sigma_x^2(1 - r^2) = \sigma_x^2 - q_2^2 \sigma_y^2 \\ s_y^2 &= \sigma_y^2(1 - r^2) = \sigma_y^2 - q_1^2 \sigma_x^2 \\ q_1 \sigma_x^2 &= q_2 \sigma_y^2. \end{aligned} \right. \tag{5}$$

Since these four equations determine relations between these seven values, there are only three that are independent. The character of the problem to be investigated determines which among the seven values may be considered as independent factors. To give an example: When we investigate proportions of the body we may recognize that in a group of common descent individuals of a certain stature have a certain average weight which has a certain variability. The relation is a physiological and anatomical one, and the stature partly determines the weight. The three values, the relation of which determines the other values are, therefore, σ_x which expresses the range of stature of the individuals examined; s the variability of the independent array of weights; and q_2 the coefficient which determines the average weight as determined by stature. All the other values must be expressed in terms of these three.

$$\sigma_y^2 = s_y^2 + q_1^2 \sigma_x^2$$

$$q_2 = \frac{q_1\sigma_x^2}{s_y^2 + q_1^2\sigma_x^2}$$

$$r = \frac{q_1\sigma_x}{\sqrt{s_y^2 + q_1^2\sigma_x^2}}$$

$$r = \frac{1}{\sqrt{1 + \frac{s_y^2}{q_1^2\sigma_x^2}}}. \tag{6}$$

If, for any reason, there is a selection of values of x, the coefficient of correlation will change its value, although the actual functional relation between x and y remains the same. In other words, in cases of this type, the coefficient of correlation is an artificial value, an algebraic convenience which depends upon the composition of the series. It follows from this that, for instance, in military statistics in which a selected group is examined, the coefficient of correlation for the selected group cannot coincide with the coefficient for the total series.

It must be remembered that in this case q_2 will no longer be constant, since the arrays are disturbed by the selection. The general theory, however, remains unchanged if we substitute in equation (2*b*)

$$[xy] = [q_2y^2] = q_2'\sigma_y^2$$

and utilize q_2' in place of q_2. It will be seen that in this case the second assumption, namely, the equality of the variability of the arrays of y, does not hold good. If there is a selection with regard to both variables, as in military statistics, when individuals of underweight and of understature are eliminated, neither q_1 nor q_2 can be determined according to the method here discussed. We must substitute for equation (4*a*),

$$\sigma_y^2 = [s_y^2] + [(q_1x)^2] \tag{7}$$

then equation (6) takes the form

$$r = \frac{1}{\sqrt{1 + \frac{[s_y^2]}{[(q_1x)^2]}}}.$$

In these cases, the essential problem will be not the determination of r, but the determination of the two values of q.

If we had a selection in which only one end of the distribution were eliminated the problem could easily be solved. In most cases, however, the cutting off of one end will affect to a greater or lesser extent the adjoining part of the distribution curve. This may be seen clearly in the distribution of statures in the Italian army as given by Livi in his

Antropometria Militare. The lowest stature is 154 c., but those just above 154 c. are also reduced in numbers because individuals of low chest measurements and other deficiencies will be more common among those individuals whose stature is a little over 154 than among tall ones. It does not seem, therefore, worth while to carry through a detailed calculation of the general problem because the characteristics of the distribution will have to be investigated in each particular case.

The dependence of the coefficient of correlation upon the distribution of the variables appears particularly in cases of tetrachoric distributions of disparate forms. We assume two alternative pairs of forms: a with the probability of occurrence p_a, and *not* a with the probability of occurrence q_a; and b and *not* b with the respective probabilities of occurrence p_b and q_b. Among all the cases of occurrence of a, b combined with a has the probability f_{ab}; among all the cases of non-occurence of a, b combined with *not* a the probability f_{ob}. In the same way, among all the cases of occurrence of b, a combined with b has the probability f'_{ab}; among all the cases of non-occurrence of b, a combined with *not* b the probability f'_{oa}.

When in a particular series the form a appears p_a+x times, the form *not* a will appear q_a-x times. In such a group of series b will occur on the average

$$[p_b+y]=f_{ab}(p_a+x)+f_{ob}(q_a-x) \text{ times; and}$$

$$[y]=x(f_{ab}-f_{ob})=cx.$$

In the same way $$[x]=y(f'_{ab}-f'_{oa})=c'y.$$

Hence $$[xy]=c[x^2]=c'[y^2]$$

since $$[x^2]=\frac{p_aq_a}{n},\quad [y^2]=\frac{p_bq_b}{n}$$

$$[xy]=c\frac{p_aq_a}{n}=c'\frac{p_bq_b}{n}$$

$$\frac{c}{n}=r\sqrt{\frac{p_bq_b}{p_aq_a}}$$

$$\frac{c'}{n}=r\sqrt{\frac{p_aq_a}{p_bq_b}}$$

$$r=\sqrt{\frac{[xy]}{p_aq_a\cdot p_bq_b}}.$$

In this case c and c' are quantities determined by the character of the series: p_a and p_b express the relative frequencies of the two types treated. It appears, therefore, that r is determined by these frequen-

cies. To give an example: Given a large bag of black (np_a) and white (nq_a) pieces of cardboard; these pieces are in part round (np_b), in part square (nq_b); $np_a f_{ab}$ pieces are black and round; $nq_a f_{ob}$ are white and round; $np_b f'_{ab}$ are round and black; and $nq_b f'_{oa}$ are square and black. Then even if the values of f remain the same, the coefficient of correlation is determined by the frequencies p_a and p_b. It will be understood that the values of f are all interdependent. We may write

$$f_{ab} = p_b + \frac{\delta}{p_a}; \qquad f'_{ab} = p_a + \frac{\delta}{p_b}$$

$$f_{ob} = p_b - \frac{\delta}{q_a}; \qquad f'_{oa} = p_a - \frac{\delta}{q_b}.$$

Then, as is easily shown, for a series of n cases

$$\delta = [xy].$$

It is obvious that this proof holds good only in cases of alternative qualities. When we have a tetrachoric distribution based on an artificial division of a range of continuous variation, the values c and c' are not constant.

I may point out that the proof given here proves the correctness of my previous discussion of this subject (*Science*, N. S. Volume 29, 1909, pp. 823–824). It does not seem to me that the criticism of the theory made by Karl Pearson (*Science*, N. S. Volume 30, 1909, p. 23) and by Karl Pearson and David Heron (*Biometrika*, Vol. 9, 1913, p. 166 and Vol. 8, 1911, p. 114) is valid. This criticism was repeated later on by Arnold Lang (*Experimentelle Vererbungslehre in der Zoologie*, seit 1900, Erste Hälfte, Jena, 1914, p. 419) on the basis of Poniatowski's remarks. I have discussed this subject more fully in a general report on *Anthropometric Investigations of the Population of the United States.*

A second case in which the purely conventional meaning of the coefficient of correlation appears clearly is that of mixed series. I have treated this problem at another place* and it may be sufficient to repeat the general argument as applied to a particular case. If we have a mixed population of round-headed and long-headed individuals who may have intermarried, and among whom there is a tendency of reversion to parental types; if, furthermore, the long heads in this series are in absolute measures long and narrow, the round heads absolutely short and wide, then short heads will, on the whole, be wide, and long heads narrow, and we find a negative correlation; while in a homogeneous series there will be a positive correlation, because both absolute measures increase with absolute size of the head. In cases of this type,

* "The Cephalic Index," *American Anthropologist*, N. S. 1: 448–461.

the coefficient of correlation is a valuable means of determining the composite character of the series. If the character of the homogeneous series is known, it may even enable us to determine the character of the composition, but it does not express the inner relation between two values.

Correlations may thus merely be expressions of the variability of a series. At another place* I have demonstrated the relation between the coefficient of correlation expressing the similarity of brothers and the variability of family groups. For large fraternities or whole populations, this relation is easily demonstrated. If a number of fraternities or local units are distributed, each around its own average, and the averages themselves are distributed according to the laws of chance, we may call the deviation of the average of a particular population or fraternity from the general average x; that of an individual in each population or fraternity from its own average y. Then, the correlation will depend upon the products

$$(x+y)(x+y^1)$$

which have to be taken in each group. Since the individuals in each group are independent

$$[(x+y)(x+y^1)] = [x^2] = \sigma_x{}^2$$

and

$$r = \frac{\sigma^2{}_x}{\sigma_x{}^2+\sigma_y{}^2}.$$

In other words, the coefficient of correlation is determined by the variability of the averages of the separate groups and that of the individuals in each group.

* "On the variety of Lines of Descent represented in a Population," *American Anthropologist*, N. S. 18: 1–9.

NATIONAL CHANGES IN HEALTH AND LONGEVITY*

By Arthur Newsholme

The subject roughly indicated in the title as announced evidently cannot be even outlined in an hour's address; my remarks will deal chiefly with some of the changes in longevity in England and, so far as they are ascertainable, in this country. After a more general introduction, I propose to confine my review to men, merely indicating that improvements in the male have in nearly every instance been exceeded in the female sex. As will be seen shortly, I propose, furthermore, to limit my comparisons chiefly to the period beyond the 40th year of human life.

It will be agreed that the object of preventive medicine is to secure long life with enhanced health. The mere desire for long life may, from a certain angle, be regarded as including some ignoble elements. There is a sense in which we must agree with the Wisdom of Solomon:

"Honorable age is not that which standeth in length of time, nor is measured by number of years. He being made perfect in a short time fulfilled a long time."

Although it is the desire of every man to die of old age, there are many passions more powerful. As Lord Bacon said:

"There is no passion in the mind of man so weak, but it mates and masters the fear of death. . . . Revenge triumphs over death; love slights it; honor aspires to it; grief flyeth to it; fear pre-occupateth it."

But to die of old age is the laudable ambition of all. We may assume that death is a normal, though the last, act of life. In Goethe's words, "Life is the most exquisite invention of nature, and death is her expert contrivance to get plenty of life."

To die of age is comparatively rare. Disease and accident, in times of peace, are the chief causes of death in civilized communities; premature and therefore wasteful death is the rule among us. This is illustrated by the following table, which shows in various life-table experiences the age at which a given number (say 100,000) starting at birth becomes reduced to half its original size (say 50,000).

It is convenient here to explain for the benefit of non-technical readers that a life table represents "a generation of individuals passing through time." Theoretically, it may be formed by actually watching

*A lecture delivered before the Harvey Society, Jan. 29, 1921.

a large group of persons from birth to death, and ascertaining the number of survivors and the average future expectation of life at each successive birthday. But such a life table would be obsolete before it could be utilized; and in the life-table figures quoted hereafter, the death rates for each age-period of life during a short series of years are assumed to determine the number of survivors to the next age-period, who are then subject to the death rate of the next age-period in the same years, and so on.

TABLE I

AGE AT WHICH POPULATION IS REDUCED TO ONE-HALF POPULATION AT BIRTH (LIFE-TABLE EXPERIENCE)

	Experience of	Males	Females
England and Wales	1901–10	58–59 years	62–63 years
" " "	1909–11	61–62 "	65–66 "
United States of America:			
Experience of	1909–11		
Entire population	Original registration states	58–59 years	62–63 years
White	"	59–60 "	63–64 "
Negro	"	34–35 "	40–41 "
Native white	"	60–61 "	64–65 "
White in cities	"	55–56 "	60–61 "
White, rural	"	65–66 "	67–68 "
Indiana		64–65 "	66–67 "
Massachusetts		58–59 "	62–63 "
Michigan		64–65 "	66–67 "
New Jersey		57–58 "	62–63 "
New York		55–56 "	61–62 "

Dr. Farr (35th Ann. Rep. of Reg. General) described the age between 45 and 55 as the middle arch of life, since shortly after the 45th year of age, a million born at the same time were reduced to half a million. Now a much larger proportion pass this middle arch of life.

In most of the communities mentioned, half of the total population born has scarcely disappeared before reaching the 60th annual turnstile.

This is true for a life-table population in which a given number of persons are traced through life on a fixed basis of experience. In actual life there is in most communities a greater stream of incoming

TABLE II

ENGLAND AND WALES, 1901–10

PROPORTION OF DEATHS AT ALL AGES (=100) AT DIFFERENT AGE-PERIODS

Age	Males		Females	
First 5 years	35.0		31.0	
Twenty years 5–25	7.8		8.3	
" " 25–45	12.6		12.2	
" " 45–65	20.8	44.6	19.1	48.5
Ages 65 and upwards	23.8		29.4	
	100.0		100.0	

new lives by birth than of departing lives; the loss of life under these circumstances is heaviest in the earlier years of life. This is illustrated in Table II, which gives the proportion of deaths at all ages that occurred at different age-periods in England and Wales during the years 1901–10.

Thus, in actual experience only 44.6 per cent of the male and 48.5 per cent of the female deaths occur at ages over 45.

THE VITAL SUPERIORITY OF THE FEMALE

It will be noted that a smaller proportion of female than of male deaths occur in early life; also that on the average females live longer than males. The following table shows that at every age except the age between 5 and 15, the female death rate is lower than that of the male at corresponding ages.

TABLE III

PERCENTAGE EXCESS OF MALE OVER FEMALE DEATH RATE AT EACH AGE-PERIOD DURING TWO DECENNIA*

Age	1851–60	1901–10
0–	+15	+19
5–	+1	−3
10–	−3	−6
15–	−9	+7
20–	+4	+19
25–	−4	+17
35–	+3	+22
45–	+18	+30
55–	+14	+28
65–	+11	+20
75 and upwards	+6	+12

* This table and most of the other tables given in this paper are derived from Dr. Stevenson's contributions to the Reports of the Registrar General of England and Wales.

The death rate at every age in both sexes has declined, but males show an increasing excess of mortality as compared with females, amounting to a maximum excess of 30 per cent at the age-period of 45–55. The excess of female over male mortality at ages 5–15 in 1901–10 and at ages 10–15, which has held good during seven consecutive decennial periods of English experience, would form an admirable subject for further study. Except at these ages the superiority of female over male vitality is evident throughout life—a fact which is perhaps too little known.

In England and Wales, for every 1,000 live-births of female infants there were 1,049 live-births of male infants in 1841–50, the proportion slowly falling to 1,038 in 1901–10, and rising again to 1,045 to every 1,000 female births in the three years 1916–18.

In the birth registration area of the United States in 1918 the pro-

portion of male to female live-births was 1,059 to 1,000 among the white, and 1,020 to 1,000 among the negro population.

But this excess of males over females is soon removed. The superiority of the female in the struggle for survival is shown markedly in the first day after live-birth, through every week of the first month, and in each trimester of the first year of extra-uterine life; and it is shown in American and in English experience alike. By the end of the second year of extra-uterine life, girls outnumber boys. That greater facility of birth of females owing to a smaller cranium is not the sole cause is evidenced by the persistence of the phenomenon of superior vitality.

TABLE IV

DEATH RATE AT EACH PERIOD OF INFANCY PER 1,000 BIRTHS

	England and Wales, 1918		United States, birth registration area, 1918	
	Males	Females	Males	Females
Under one day	12.6	9.6	17.6	13.1
1 day and under 1 week	13.3	10.7	17.0	13.1
2nd week	6.0	4.9	6.5	5.4
3rd "	5.2	3.8	4.8	3.9
4th "	3.9	2.8	3.7	3.1
Total under 1 month	41.0	31.8	49.6	38.6
1 to 3 months	18.9	15.0	17.1	13.7
3 to 6 "	18.1	13.6	17.9	14.5
6 to 9 "	15.6	12.6	14.5	12.7
9 to 12 "	14.3	13.0	11.7	11.1
	107.9	86.0	110.8	90.6

DEATH RATES AT DIFFERENT AGES: INCREASES AND DECREASES

For the registration area of the United States, comparisons are possible only between 1900 and subsequent years. In Table V the American experiences for the single years 1900 and 1911 are quoted from the Mortality Statistics, 1911 (Bureau of Census), page 22. The death rates for the same years and ages are given for England and Wales.

TABLE V

MALES: DEATH RATE PER 1,000 LIVING AT EACH AGE

Year	0–	5–	10–	15–	20–	25–	35–	45–	55–	65–	75 and over
1900 England and Wales	61.6	4.2	2.3	3.7	5.1	6.7	11.7	19.9	37.1	74.2	168.3
United States reg. area	54.2	4.7	2.9	4.9	7.0	8.3	10.8	15.8	28.9	59.6	146.1
1911 England and Wales	46.2	3.5	2.0	3.0	3.9	5.0	8.0	14.8	29.7	63.5	150.4
United States reg. area	39.8	3.4	2.4	3.7	5.3	6.7	10.4	16.1	30.9	61.6	147.4

The differences are interesting, but it is desirable to compare the experience of other years before final conclusions are drawn. In 1900, at ages 0–5 and at all ages over 35, registration America had a more favorable male death rate than England; in 1911 its superiority at higher ages was confined to ages over 65. In 1900 the English death rate at ages 0–5 was 14 per cent and in 1911 was 16 per cent higher than that of America (registration area). Evidently, then, in view of the higher infant mortality in America, the death rate at ages over 1 and under 5 is lower in the United States than in England, a subject worthy of further study. For all ages the standardized death rate of England and Wales was 19.9 per 1,000 population in 1900, and 15.6 in 1911; in the original registration area of the United States it was 17.6 in 1900, and 15.3 in 1911; there being a 22 per cent improvement in England, and a 13 per cent improvement in the registration area.

THE INCIDENCE OF REDUCED DEATH RATE AT DIFFERENT AGES

In the next table is shown the historical trend of the death rates of England and Wales in five successive decennial periods, the increase or decrease of the death rate for various age groups being displayed.

TABLE VI

INCREASE OR DECREASE PER CENT OF THE DEATH RATE FOR EACH SEX AND AGE-GROUP COMPARED WITH THE DEATH RATE FOR THE SAME GROUP IN THE IMMEDIATELY PRECEDING DECENNIUM

	England and Wales				United States
	1871–80 compared with 1861–70	1881–90 compared with 1871–80	1891–1900 compared with 1881–90	1901–10 compared with 1891–1900	1911 compared with 1900
Males					
under 5	−6.9	−10.0	+1.8	−20.2	−26.6
5–	−18.1	−20.3	−19.4	−18.7	−27.7
10–	−17.3	−20.4	−17.1	−16.1	−17.2
15–	−15.1	−17.8	−12.2	−18.5	−24.5
20–	−13.1	−22.1	−11.7	−17.4	−24.3
25–	−6.0	−16.9	−13.0	−17.6	−19.3
35–	+2.3	−10.2	−7.2	−20.4	−3.7
45–	+4.2	−3.5	−2.2	−14.4	+1.9
55–	+5.2	−0.5	+0.7	−9.0	+6.9
65–	+3.9	+1.1	−0.1	−7.9	+3.4
75 and over	+2.3	−3.9	−1.6	−4.8	+0.9
Females					
under 5	−8.4	−11.0	+1.6	−20.7	−27.3
5–	−20.1	−15.6	−16.9	−17.3	−32.6
10–	−17.5	−16.4	−17.5	−15.6	−32.3
15–	−17.9	−18.9	−17.1	−21.1	−31.3
20–	−14.6	−18.7	−19.5	−21.6	−29.9
25–	−11.4	−14.4	−17.5	−22.0	−26.8
35–	−3.6	−9.0	−9.4	−21.5	−15.3
45–	−0.2	−3.3	−2.4	−15.1	−9.2
55–	+2.7	−0.7	−0.1	−12.6	+0.8
65–	+3.2	−1.0	+0.5	−11.2	+2.4
75 and over	+0.8	−5.2	−1.1	−7.0	−0.2

The table shows also a similar comparison between the experience of 1911 and 1900 for the registration area of the United States.

It will be noted that the American experience displays for the later period an increased death rate in men at all ages over 45, and in women at ages 55 to 75.

When the experience of 1871–80 is contrasted with that of 1861–70, the English experience shows an increased death rate at all ages over 35 for males, and at ages over 55 for females. When 1881–90 is contrasted with 1871–80 there is no evidence of increased death rate at any age except in men aged 65–75; and the experience of the 20 years 1891 to 1910 shows a declining death rate in both sexes at all ages, with the exception of a slight increase at ages 55–65 in males and at ages 65–75 in females in 1891–1900, owing probably to the influenza pandemic of 1889–93. The evidence is definite that there has been steady advance in the age to which increasingly favorable death rates extend.

Thus, a comparison of the experience of (the death registration area of) the United States with that of England and Wales shows that the registration area stands historically in respect of increase or decrease of death rate at various stages of life approximately where England stood in 1871–80; and it is not very hazardous to make the same forecast for the United States that I ventured to make for England in the year 1893 when the first *Brighton Life Table* was prepared by me.* In that publication, I pointed out that:

"It is evident that although in England, owing to the large number of lives saved during the early years of life, the number surviving to the higher ages has increased, thus securing a great gain to the community, this is not incompatible with a stationary or even diminished prospect of life for each individual over a certain age. In England the death rate for males was higher in 1871–80 for *all age-groups above the 25-35 period*, and for females was higher in 1871–80 for all age-groups above the *35–45 period* than in preceding decennial periods."

After discussing the influence of increased wages and improved nutrition in more than counterbalancing the unfavorable influence of city conditions of life, I laid stress on the following factor which must, I believe, be given a large share of the credit for the reduced death rate at older as well as at younger ages which has now been realized.

"Another consideration requires to be borne in mind. We are at present in a transition period. The Public Health Acts of 1871 and 1875 heralded immense improvements in sanitation, the fruits of which have not yet been fully reaped. There has been, more especially since 1875, steady and increasing improvements in the conditions under

*See also p. 316 of the author's *Elements of Vital Statistics*, 1899.

which people live. Men now 40 years of age were born in the pre-sanitary period; and the first 20 years of their life were spent under more unhygienic conditions than those now holding good. This fact would go far towards explaining a stationary death rate at the higher ages. It does not, however, explain an increased death rate at those ages.

The explanation of this increased death rate at the higher ages will probably be evident when at the end of another 20 or 30 years the improved conditions of life have endured sufficiently long to enable their full force and value to be determined. We must be content in the meantime to have stated the more important factors which appear to be at work, leaving the complete solution of the problem to a time when the statistical experience of our country is more mature."

At the end of the time asked for in the foregoing comment, it is noteworthy that at every age, even beyond 75 years of age, the number dying for a given number at risk has decreased; and the improved conditions of modern life, sanitary, social, and economic, have resulted in England in a lowered death rate at every recorded age-period.

It is unfortunate that a similar comparison can be made for the registration area of the United States only for 1900 and 1911; which, as already stated, displays a difference similar to the one displayed when the English death rates at different ages in 1871–80 are compared with corresponding death rates for 1861–70.*

CHANGES IN ADULT MORTALITY

Having given a brief outline of the general lowering of the death rate at every period of life and having shown the greater share of women in this improvement, I propose now to confine my observations to the historical changes after the age 25, and more particularly to events after the 40th milestone of life has been passed, as it is especially concerning this period of life that our professional pessimists—as always in the past—insist that we are "going to the dogs."

* This statement gives the historical position. It will be noted, however, that the standardized total American death rate in 1911 was 15.3 as compared with 15.6 for England and Wales. I am able to give the following additional contribution to the historical position. In the annual report of the Massachusetts Board of Health for 1896 is given a comparison of the death rate according to age for both sexes in combination in 1875 and 1895 respectively.

DEATH RATE, MASSACHUSETTS, U. S. A.

	0–	5–	10–	15–	20–	30–	40–	50–	60–	70–	80–
1875.....	73.9	9.8	4.7	7.7	10.5	11.3	13.0	18.3	34.8	71.1	176.4
1895.....	64.5	6.2	3.2	5.3	7.1	9.7	12.7	20.5	39.4	82.4	184.7

It will be noted that the death rate had increased between 1875 and 1895 at each age-period after 50. A somewhat similar increase has occurred between 1900 and 1911.

It is particularly unfortunate that these historical comparisons, owing to the absence of comparable American data, must for the present be limited chiefly to English experience with a relatively stable population and long experience of accurate vital statistics. For England and Wales as a whole, a consecutive series of life tables makes possible comparisons for a long series of years. In the following tables the facts are shown for males.

TABLE VII

LIFE TABLES FOR MALES

A—NUMBER OF SURVIVORS AT HIGHER AGES, OUT OF 1,000 BORN

Experience of	England and Wales				Massachusetts, U. S. A.	
	1838–54	1881–90	1901–10	1910–12	1893–97	1909–11
No. at birth	1,000	1,000	1,000	1,000	1,000	1,000
No. at age						
25	624	694	745	779	680	760
30	595	669	727	762	651	739
35	564	640	705	742	620	713
40	532	605	677	717	588	683
45	496	564	642	685	554	646
50	456	518	599	643	515	604
55	409	463	544	590	468	551
60	356	398	476	521	411	482
65	295	322	393	435	344	395
70	223	239	299	334	267	296
75	148	154	198	224	185	197

B—NUMBER OF SURVIVORS AT HIGHER AGES OUT OF 1,000 AT AGE 25

	England and Wales				Massachusetts, U. S. A.	
No. at age						
25	1,000	1,000	1,000	1,000	1,000	1,000
30	954	964	976	978	958	972
35	904	922	946	952	912	938
40	852	872	909	920	865	899
45	794	813	862	879	815	850
50	731	746	804	826	758	795
55	655	667	730	758	688	725
60	570	574	639	669	604	634
65	473	464	528	558	506	520
70	357	344	401	429	393	389
75	237	222	266	288	272	259

C—NUMBER OF SURVIVORS AT HIGHER AGES OUT OF 1,000 AT AGE 40

	England and Wales				Massachusetts, U. S. A.	
No. at age						
40	1,000	1,000	1,000	1,000	1,000	1,000
45	932	932	948	956	942	946
50	857	856	885	897	876	884
55	769	765	804	823	796	807
60	669	658	703	727	699	706
65	556	532	580	606	585	578
70	420	395	442	466	454	433
75	278	255	291	312	315	288

Table VII A shows that in each successive life table the number of survivors to the age 25, out of equal numbers born, has continuously increased with lapse of time. The subsequent course of events for adult life can more accurately be followed in Table VII B. It will be seen that with the exception of ages 60 and upwards, in the decennium 1881–90, the number of survivors out of every 1,000 reaching the age of 25 years has steadily increased until after the Psalmist's term of life is passed. Similarly, out of every 1,000 reaching 40 years of age (Table VII C), with the exception of ages 55 and upwards, in 1881–90, improvement in expectation of life is shown at all ages over 40. These tables give no evidence of national deterioration at higher ages. They show, on the contrary, increased longevity in each succeeding period. This is further illustrated in Figure I.

For the United States death registration area* historical comparisons are practicable only for 1900 onward, except for Massachusetts, for which Dr. S. W. Abbott, the secretary of the then State Board of Health, constructed a life table based on the experience of the years 1893–97,† the data for which are probably comparable with those for Massachusetts published in the life-table volume of the Federal Census Bureau. In this volume is given *inter alia* a life table for Massachusetts, based on the experience of 1909–11; and in Table VII these two experiences are compared.

When the whole of life is taken into account the results in Massachusetts show improved prospects of life in every age-period; so in the main do the figures for equal numbers starting at age 25; but for equal numbers starting at age 40, there appears to be slight vital deterioration from age 65 and upwards.

Although the material for historical comparisons in the United States is scanty, the series of life tables issued by the Bureau of the Census, based on the deaths in the three years 1909–11 and the population of 1910, gives valuable material for almost contemporaneous comparison with the data for England and Wales. Dr. William H. Davis, the chief statistician for vital statistics of the Census Bureau, informs me that the English and the American life tables are comparable with respect to methods of construction. In Table VIII are compared the number of survivors out of 1,000 males at birth, at age 25, and in each successive five additional years of age. The columns are arranged in order of the number of survivors to the highest age, beginning with the experience of greatest vitality.

* This area comprised in 1900, 40.5 per cent of the total population of the United States.

† The 30th Annual Report of the State Board of Health of Massachusetts for 1898 gives also an earlier life table *for persons* in Massachusetts dealing with the experience of the year 1855.

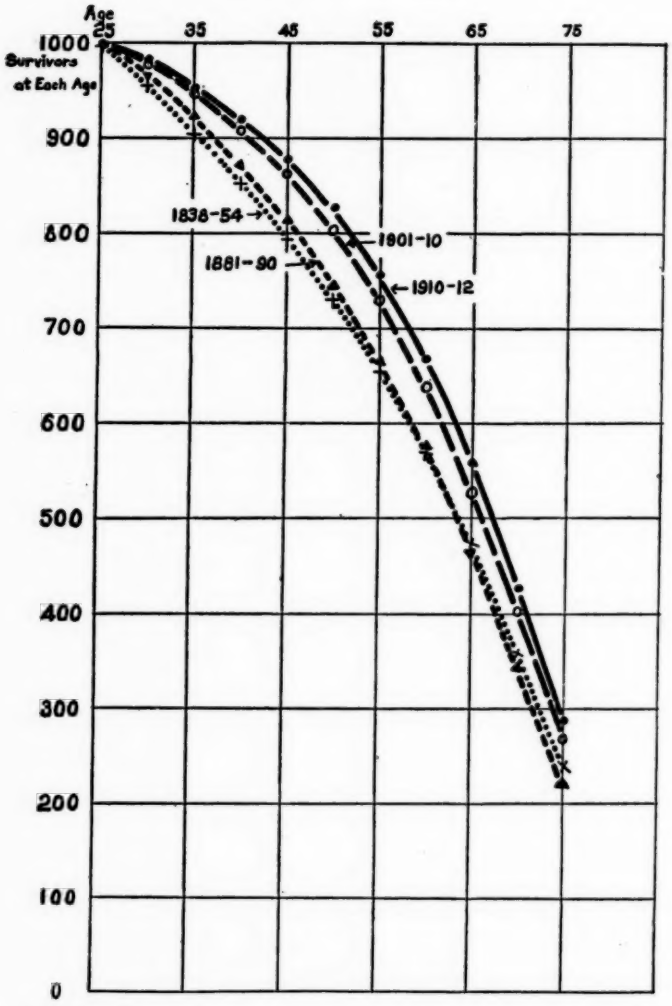


FIGURE I. LIFE TABLE EXPERIENCE, ENGLAND AND WALES, FOR FOUR PERIODS. Number of male survivors at each subsequent age out of 1,000 at age 25, in 1838–54, 1881–90, 1901–10, and 1910–12.

It will be noted that the aggregate white males in the rural parts and in the whole of the registration area, and the population of the states of Indiana and Michigan showed the largest number of survivors to age 25, Michigan coming next.*

In Figure II the relative experience of urban and rural populations and of Massachusetts is shown.

* Separate life tables have been published for Indiana, Massachusetts, Michigan, New Jersey, and New York.

TABLE VIII

UNITED STATES LIFE TABLES, 1909–11, FOR MALES

A—NUMBER OF SURVIVORS AT HIGHER AGES OUT OF 1,000 BORN

	Total white males (rural parts)*	Native-born white males*	Total white males*	White males in cities*	Indiana	Michigan	New Jersey	New York	Massachusetts
No. at birth.	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
No. at age									
25.....	806	768	770	751	804	791	764	756	760
30.....	785	744	748	728	782	771	741	731	739
35.....	763	716	721	698	757	749	712	701	713
40.....	738	683	688	662	732	725	677	662	683
45.....	710	648	651	619	702	698	637	619	646
50.....	677	608	607	569	667	663	591	569	604
55.....	637	561	556	510	628	621	537	511	551
60.....	581	501	490	435	570	565	466	440	482
65.....	506	426	409	347	498	489	381	358	395
70.....	409	337	315	253	404	393	288	269	296
75.....	294	237	216	162	290	279	194	180	197

B—NUMBER OF SURVIVORS AT HIGHER AGES OUT OF 1,000 AT AGE 25

No. at age 25.....	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
30.....	974	969	971	969	973	975	970	967	972
35.....	946	932	936	929	942	947	932	927	938
40.....	916	889	894	882	911	917	886	876	899
45.....	881	844	846	824	873	882	834	819	850
50.....	840	792	788	758	830	838	774	753	795
55.....	790	731	722	679	781	785	703	676	725
60.....	721	652	636	579	709	714	610	582	634
65.....	628	555	531	462	619	618	499	474	520
70.....	507	439	409	337	503	497	377	356	389
75.....	365	308	280	216	361	353	254	238	259

C—NUMBER OF SURVIVORS AT HIGHER AGES OUT OF 1,000 AT AGE 40

No. at age 40.....	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
45.....	962	949	946	935	959	963	941	934	946
50.....	917	890	882	866	911	915	873	859	884
55.....	863	821	808	770	858	857	793	772	807
60.....	787	734	712	657	779	779	688	664	706
65.....	686	624	595	524	680	675	563	540	578
70.....	554	493	458	382	552	542	426	406	433
75.....	398	347	314	245	396	385	287	272	288

* In the original registration states.

The picture is not materially changed when the survivorship of 1,000 men starting at age 40 in each of these states is contrasted. The populations living preponderantly in cities evidently occupy an inferior position. That the conditions of city life explain, largely at least, the marked differences of vitality in the different states is suggested by the partial coincidence between degree of urbanization and paucity of survivors. At the census of 1910, 92.9 per cent of the population in Massachusetts was urban (*i. e.*, lived in districts with a population exceeding 2,500); 78.9 per cent in New York and 75.2 per cent in New

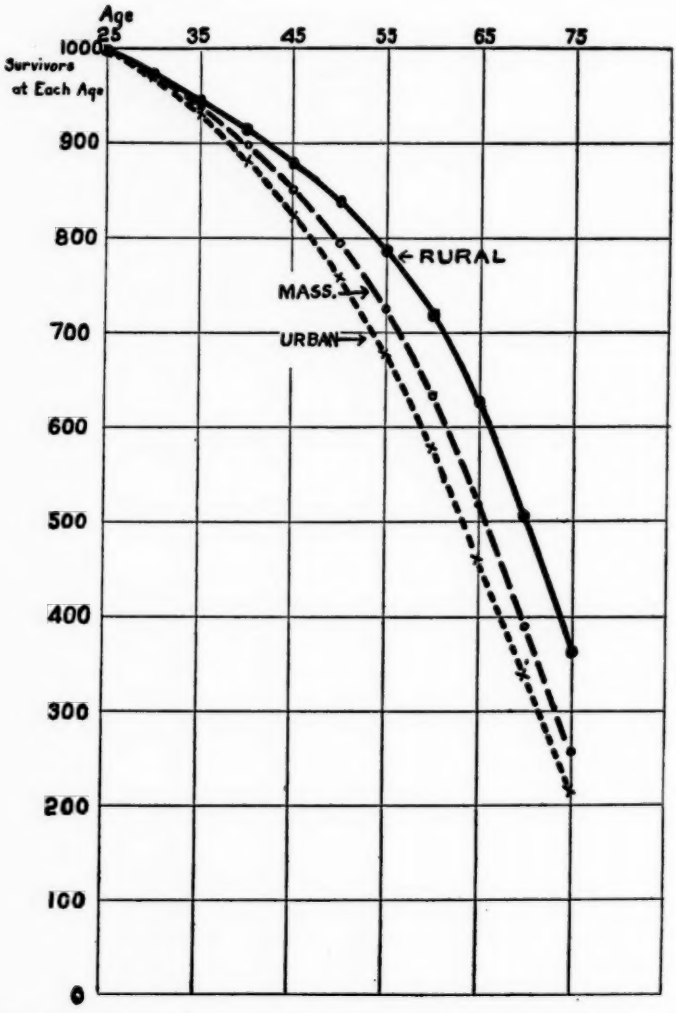


FIGURE II. LIFE TABLE EXPERIENCE OF THE UNITED STATES (1909–11). Number of male survivors at each subsequent age out of 1,000 at age 25 in the aggregate urban and rural white populations of the original registration states and in Massachusetts.

Jersey, as against 47.2 per cent in Michigan and 42.5 per cent in Indiana. The proportion of foreign-born population in 1910 was about 32 per cent in Massachusetts and New York State; 29.5 per cent in New Jersey; 22.1 per cent in Michigan; and 9.2 per cent in Indiana. It may be added that for every 100 foreign-born persons in Massachusetts the highest proportion is Irish (21.2 per cent); in New York

the greatest number are Russians (20.5 per cent), followed by Italians (17.5 per cent) and Germans (16.0 per cent).

If the experiences from age 25 onward be compared, rural males, the males of Michigan and of Indiana have approximately an equal number of survivors to the age 70, while Massachusetts, New Jersey, and the State of New York have relatively few.

Table IX compares the experience of males in England and Wales in 1910–12 with that of white males and of white native-born males in the registration area of the United States in 1909–11.

TABLE IX

A—NUMBER OF SURVIVORS AT HIGHER AGES OUT OF 1,000 BORN

	England and Wales	United States* 1909–11	
	Males 1910–12	All native-born white males	All white males
No. at birth	1,000	1,000	1,000
No. at age			
25	779	768	771
30	762	744	748
35	742	716	721
40	717	683	688
45	658	648	651
50	643	608	607
55	590	561	556
60	521	501	490
65	435	426	409
70	334	337	315
75	224	237	216

B—NUMBER OF SURVIVORS AT HIGHER AGES OUT OF 1,000 AT AGE 25

No. at age			
25	1,000	1,000	1,000
30	978	969	971
35	952	932	936
40	920	889	894
45	879	844	846
50	826	792	788
55	758	731	722
60	669	652	636
65	558	555	531
70	429	439	409
75	288	308	280

C—NUMBER OF SURVIVORS AT HIGHER AGES OUT OF 1,000 AT AGE 40

No. at age			
40	1,000	1,000	1,000
45	956	949	946
50	897	890	882
55	823	821	808
60	727	734	712
65	606	624	595
70	466	493	458
75	312	347	314

* Original registration states.

SUMMARY

The close similarity between the experience of the male population in adult life in England and in the United States is striking. It is, further, noteworthy that the inclusion of foreign with native whites does not result in a very marked change in survivorship at the higher ages.

Summing up the teaching of the preceding tables we arrive at the following conclusions:

1. The vital experiences of England and Wales and of the death registration area of the United States are very similar, and almost equal.

2. At nearly all ages, including the first year of life, females have an intenser inherent power of survival than males; and females have profited more than males from the increased vitality at all ages in recent years.

3. English experience shows in successive decennial periods a reduction of death rate which encroaches on the higher ages with advance of time. After the end of the decennium 1881–90, with an insignificant exception, each age up to the end of life has shared in the reduced rate of mortality.

4. Stated in terms of survivorship, out of every 1,000 reaching 40 years of age, improvement in expectation of life (depending on the summation of survivors at all higher ages) is shown at all ages over 40, with the exception of ages over 55 in 1881–90.

5. This temporarily increased death rate at higher ages, which was probably associated with the influenza epidemic of 1889–93, has been followed by a marked decline of death rate at these higher ages.

6. In English experience the satisfactory result is shown of a diminishing death rate at all ages, though on a much greater scale in the younger than in the more advanced years of life.

7. A comparison of the life-table experiences of Massachusetts at an interval of twelve to fifteen years gives no evidence of vital improvement.

8. The current life-table experience of the white population of the United States (registration area) is almost identical with that of England; the male population (whites) of both countries have an almost equal chance of survival to the age 25; and for equal numbers starting at age 25 or at age 40, the native-born white population of America has a larger proportion of survivors to the age 75 than the population of England.

9. The position of the negro is lamentable, assuming that in the registration area of the United States the figures are trustworthy.*

* One thousand male negroes are reduced to one-half between their 34th and 35th birthday, while this does not happen for the white population before the 59th birthday is reached. I doubt whether

10. For the rural parts of the American population the expectation of life and the number of survivors to age 75 are much higher than in the entire registration area, while in cities and in the three states (New Jersey, New York, and Massachusetts) having a preponderantly city population, life is considerably curtailed.

INCREASE IN TOTAL DEATH RATE IN THE UNITED STATES AT AGES OVER 45

Three facts emerge prominently from the above study of American and English vital statistics: first, that in England there has for several decades been an almost continuous fall in the total death rate, affecting middle and advanced as well as early life; second, that the average positions of the white population of the United States (registration area) and of that of England are approximately equal; although, third, it appears that the male death rate at ages over 45 in the United States is still slightly increasing beyond what it was eleven years earlier. Why is this?

An explanation has already been suggested, if it be admitted that time is needed for the results of almost inseparable social and sanitary improvements to be felt at all ages. If, notwithstanding her vital position equal to that of England (owing, possibly, to the higher wages, better nutrition, and less alcoholism of the mass of the American as compared with the mass of the English population), she is still historically in the position as to age distribution of death rate occupied by England in 1871–80, when the older portion of the English population had not enjoyed in their childhood the social-sanitary betterment which their children enjoyed, then we may anticipate an extension of the reduced death rate in the United States to all ages ere long.

This view of the reason for the failure of the American population at the higher ages to share in the reduced death rates of earlier life is confirmed by the complex position of this country with respect to immigration. As already seen, nearly one-third of the population of the states of Massachusetts, New York, and New Jersey are foreign-born, and of the remaining two-thirds about half have foreign parent-

this figure can be trusted. Deficient birth registration among the negroes may have vitiated to some extent the life table estimates of negro population at ages 0–5, and increased the apparent death rate during this period. This would not, however, influence the number of survivors in a life-table population out of 1,000 males who reach age 25 as shown in the following table:

	Age										
	25	30	35	40	45	50	55	60	65	70	75
Total white pop. reg. area United States..	1,000	971	936	894	846	788	722	636	531	409	280
Negroes reg. area.....	1,000	938	865	787	704	614	516	413	309	213	130

age or mixed foreign and native parentage. These foreigners have come in varying proportions from Germany, Ireland, Italy, Russia, and Austria-Hungary, and in many cases they have in earlier life been constantly exposed to circumstances of malnutrition and insanitation; and, furthermore, in the earlier years of their residence in the United States they possibly have been subjected to excessive strain and privation in insanitary surroundings. It would be surprising if there were not a persistently high or even an increased death rate of persons at higher ages in the registration area. This subject has been elucidated statistically by Dr. Louis I. Dublin, who has shown that the foreign-born in New York and Pennsylvania experience a higher death rate than native-born of native parentage at nearly all older ages, and that this holds good also for native-born of foreign or mixed parentage.*

THE REGISTERED INCREASED MORTALITY FROM SPECIAL DISEASES

Having arrived thus far, we are now able to consider the statements frequently made as to special causes of alleged increased mortality at higher ages. The following statement, based doubtless on official published figures, may be quoted as typical of numerous similar statements published in scientific journals and in the daily press.

"The death rate from degenerative diseases in the United States registration area has increased 41 per cent in 20 years. By this term is meant the wear-and-tear diseases such as cancer, arteriosclerosis, Bright's disease, etc., which are due to bad personal habits."

In examining the question as to whether this increase is real or apparent, or only partially real, we should remember the limitations of accuracy of medical certification of death. There has been steady improvement in medical certification, but this in itself has necessitated caution in accepting historical comparisons of diseases. The rule of safety in making historical comparisons is to confine such comparisons to individual diseases and not combine them into groups, as "circulatory," "renal," "nervous," and so on. The same rule applies, though to a lesser extent, when making contemporaneous comparisons. If I depart from this counsel of wisdom in what follows, it is to illustrate the dubiety attaching to the statistics thus displayed.

PNEUMONIA AND BRONCHITIS

Even for diseases like pneumonia and bronchitis there is some ambiguity in historical comparisons of statistics. There is not only change in medical fashions of certification, but also in one period a larger share of the pneumonia may be secondary to uncertified influenza, or of bronchitis or pneumonia to uncertified measles, than in another period. In

* "The Mortality of Race Stocks in Pennsylvania and New York," by L. I. Dublin and G. W. Baker. *Quar. Publ. of the Amer. Statist. Assoc.*, March, 1920.

the following curves the death rates from pneumonia and bronchitis in England and Wales among males at various ages over 25 for the two periods 1881–90 and 1901–10 are compared.

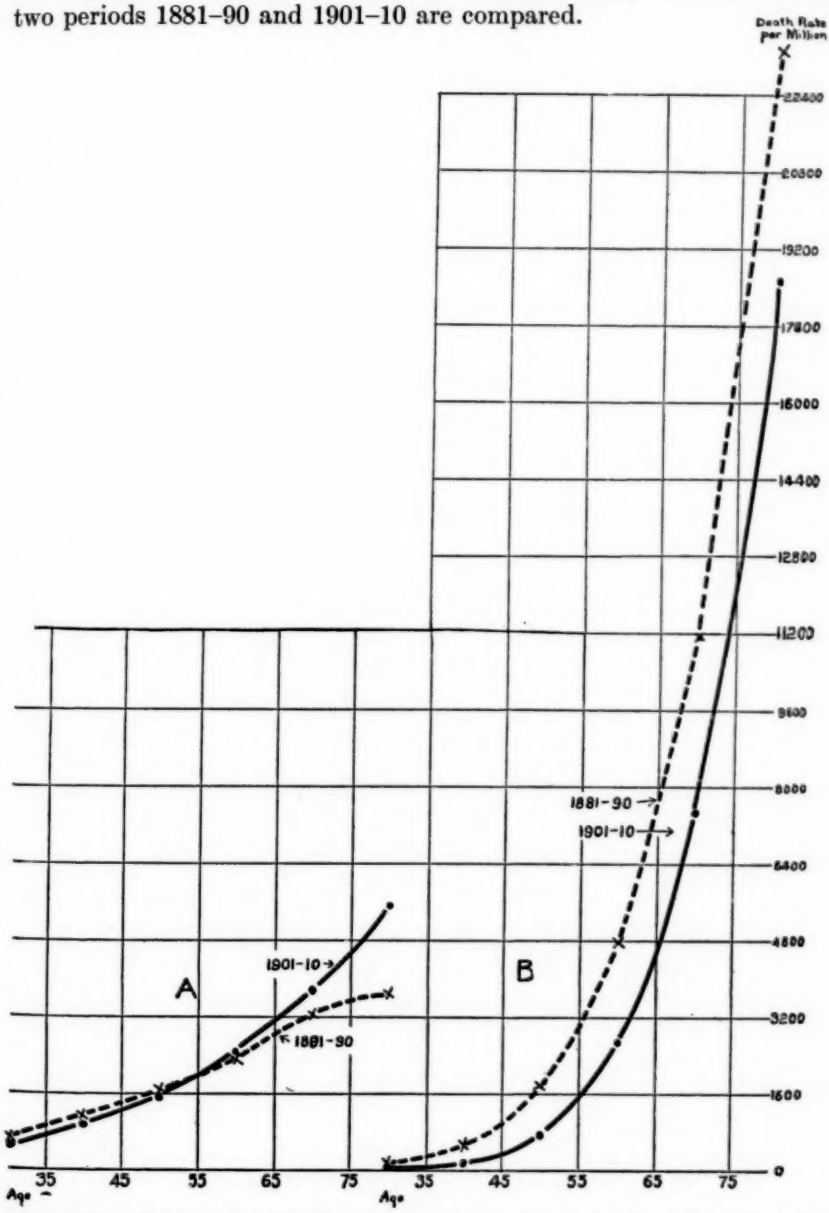


FIGURE III. DEATH RATE PER MILLION MALES AT EACH AGE PERIOD IN 1881–90 AND 1901–10 from

A. Pneumonia. B. Bronchitis.

The decline in the male death rate from bronchitis at each age-group is noteworthy, as is also the fact that in the decennium 1901–10 the death rate from pneumonia increased only at ages 65 and upwards. Notwithstanding the increased longevity of the population in the year 1918 in England and Wales, only 3.8 per cent of the total male deaths was attributed to old age or senile decay, while in the 25 years 1848–72 the number under this heading averaged 5.1 per cent of the total deaths. Even more striking as showing changes in medical certification is the following illustration of American experience, borrowed from a paper by Dr. L. I. Dublin on "The Registration of Vital Statistics and Good Business."

CARDIAC AND RENAL DISEASES

These considerations apply with even greater force when historical comparisons are made of the mortality from cardiac and renal diseases, as shown for the English experiences in 1881–90 and 1901–10 respectively (Figure V). After age 45 an increase is shown in the death rate (stated in terms of the population at the ages under risk) for each group of diseases, the maximum increase being at ages 65–75 and at 75 and upwards. To what extent this increase is real, and to what extent it arises from more accurate medical diagnosis and certification of the cause of death, cannot be stated with any degree of probability; but it must be borne in mind that this increase is associated with a reduction in the death rate from all causes in the aggregate at these very ages. The only absolutely certain facts are that on the average in English experience we live longer than in the past; but such as do not die literally of old age are certified in a larger proportion than in the past as having died of circulatory and renal diseases.

TUBERCULOSIS

The curves of English experience in 1881–90 and in 1901–10 for tuberculosis (Figure VI) show the decline in the death rate from this supremely important disease; also the gradual postponement of the age of maximum death-toll from it. We are more immediately concerned in this paper with mortality after middle life, and the curves bring out the too little remembered fact that tuberculosis remains one of the chief causes of death right up to old age.

CANCER

It would require a separate lecture to discuss the question of the increase of the registered death rate for cancer (shown in Figure VI).

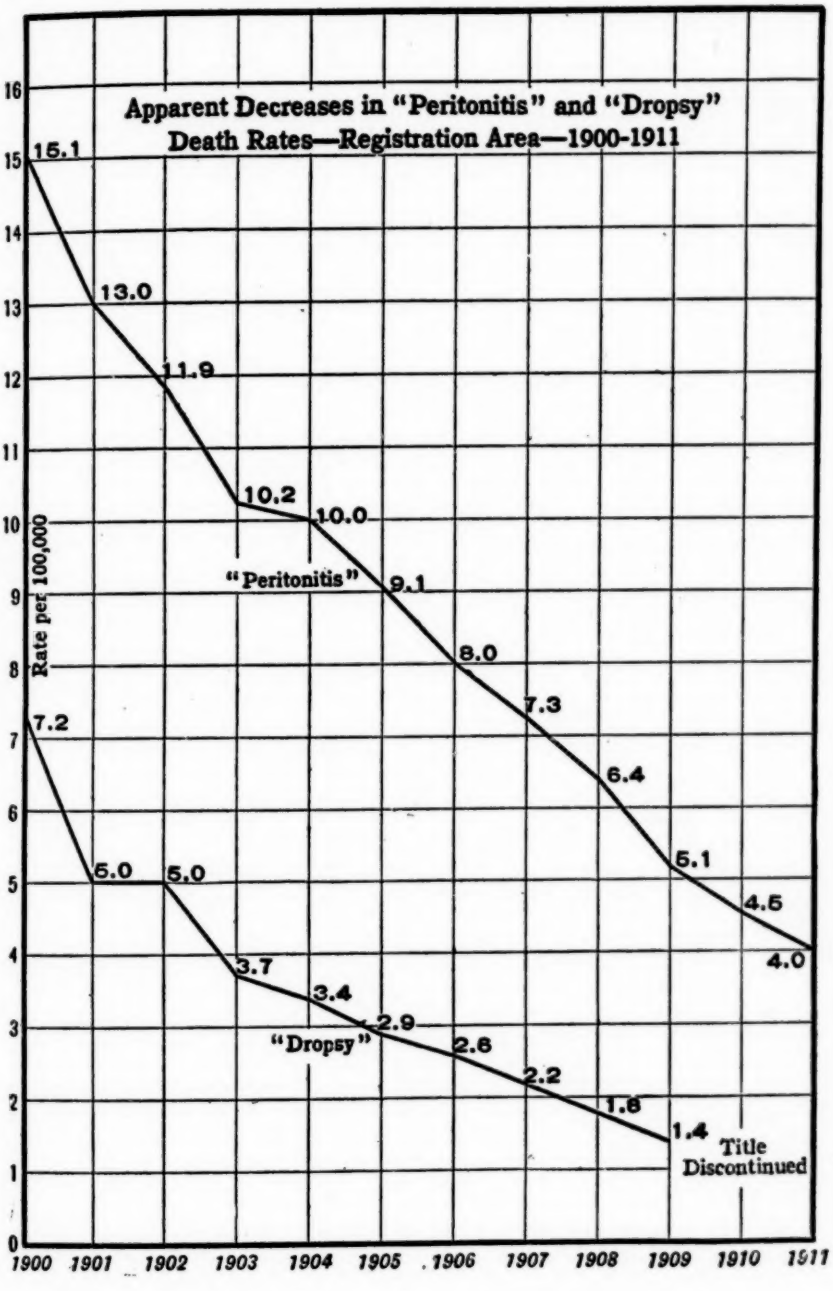


FIGURE IV. ILLUSTRATING TRANSFERENCE OF CAUSES OF DEATH

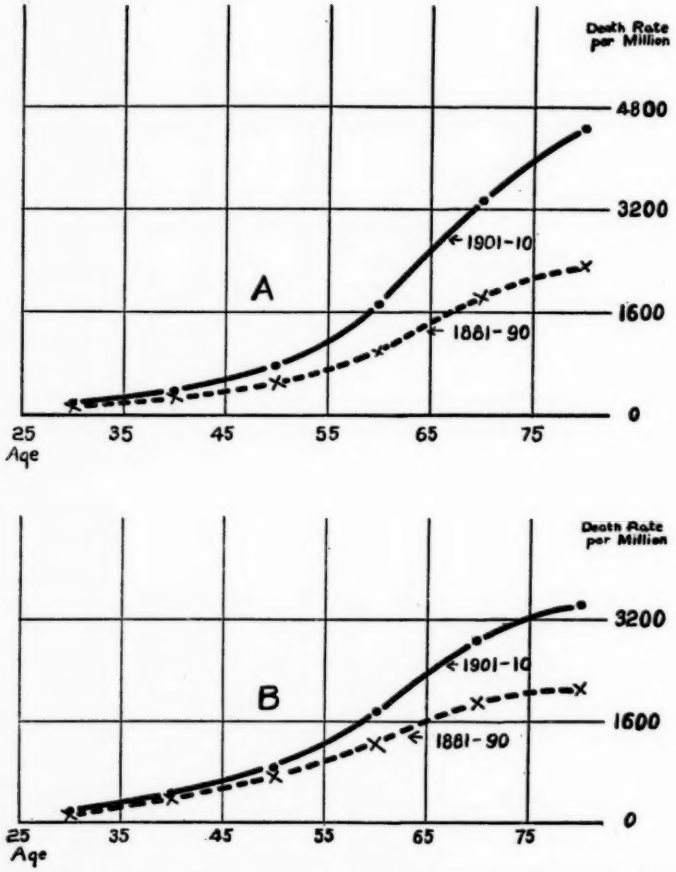


FIGURE V. DEATH RATE PER MILLION MALES AT EACH AGE PERIOD OVER 25 IN 1881–90 AND 1901–10.

A. Valvular disease of the heart, endocarditis and angina pectoris.

B. Acute and chronic nephritis, Bright's disease.

The balance of evidence appears now to support the view that cancer of certain organs has increased, although Professor W. F. Willcox, bringing up to date a paper by Mr. George King, F. I. A., and myself,* has recently come to the conclusion tentatively reached by us in 1893.

In this earlier paper the national returns for each division of the United Kingdom were subjected to accurate analysis, and statistics were given based on the death rate of Frankfort-on-Main for 1860–89, showing the incidence of cancer according to site. The conclusion

* "On the Alleged Increase of Cancer," Proc. Roy. Soc., Vol. 54, 1893.

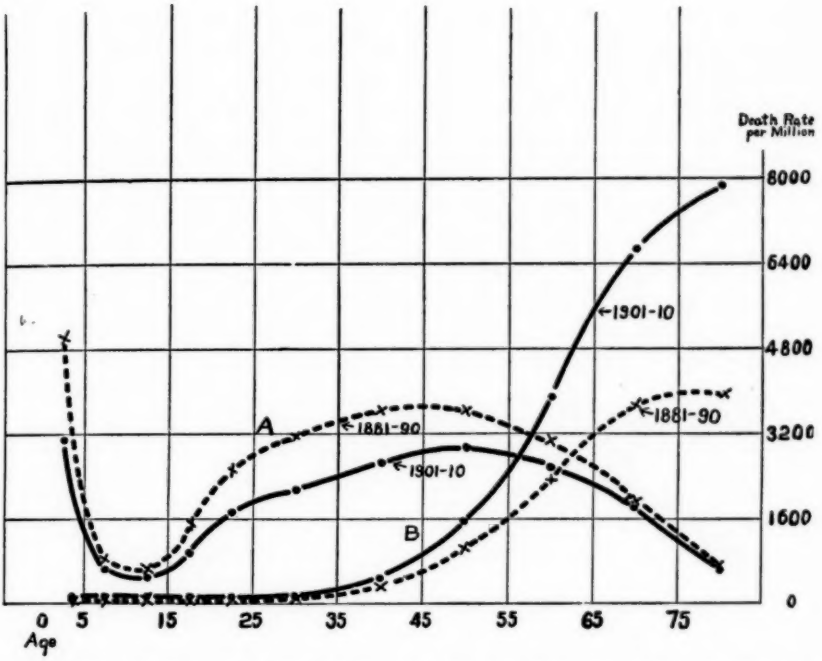


FIGURE VI. DEATH RATE PER MILLION MALES AT DIFFERENT AGE PERIODS IN 1881–90 AND 1901–10.

A. Tuberculosis (all forms).

B. Cancer.

reached was that the apparent increase of cancer is confined to "inaccessible" cancer of difficult diagnosis; and Professor Willcox has continued his comparison of cancer death rates in Frankfort down to 1913 with the result of confirming the earlier observations. The subject has been rediscussed in much detail by Dr. T. H. C. Stevenson in the annual report of the English Registrar General for 1917, in the light of English cancer data giving localization of the lesion; the conclusion he reaches is that in England "amongst males mortality from accessible cancer has increased more rapidly than from inaccessible, whereas amongst females the position is reversed, the result for both sexes jointly being a moderate excess of increase from inaccessible cancer."

In other words, the English figures do not support the conclusion drawn from the Frankfort figures, which at the time they were examined were the only figures available. Whether cancer mortality is increasing or not, the practical point is that from middle life onward it is one of the chief causes of premature mortality.

So far we have seen that a remarkable increase has occurred in the registered death rate at ages over 45 from certain diseases; and that this is associated with a declining death rate from all causes in the aggregate at these ages. The absence of a corresponding decline in the total death rate at these higher ages in the registration area of the United States is better explained by immigration of a diverse population than by the assumption that causes of degeneration are operating in the native-born population of the United States to an extent beyond that in England. Of course this does not imply that the conditions causing "degenerative diseases" do not require appropriate action in both countries. The English Registrar General in his *Decennial Supplement for 1901–10*, gives valuable tables of the death rate at various ages from 32 of the chief causes of death. From these tables the diagrams already given comparing the experience of 1881–90 and of 1901–10 for pneumonia, bronchitis, tuberculosis, cancer, and cardiac and renal diseases have been taken.

In the twenty years between the two periods the chief causes of death which showed decrease and increase respectively were as follows:

CAUSES OF DEATH

Group I. Decreasing	Group II. Increasing
Smallpox	Influenza
Measles	Diphtheria
Scarlet fever	Diarrheal diseases
Whooping cough	Pneumonia
Croup	Cancer
Enteric fever	Diabetes mellitus
Syphilis	Valvular diseases of heart endocarditis
Tuberculosis	Nephritis and Bright's disease
Rheumatic fever and rheumatism of the heart	Suicide
Meningitis	
Epilepsy	
Laryngitis	
Bronchitis	
Pleurisy	
Violence	

To supplement the evidence already stated, the following diagram may be studied. In A is shown the age distribution of the male death rate from all the diseases in Group I (above) in 1881–90 and in 1901–10 respectively. In B are displayed the corresponding facts for Group II; while in C is shown the age-distribution of the total male death rate from all other causes in the two decennia.

We thus see, as has already been displayed by the life-table method,

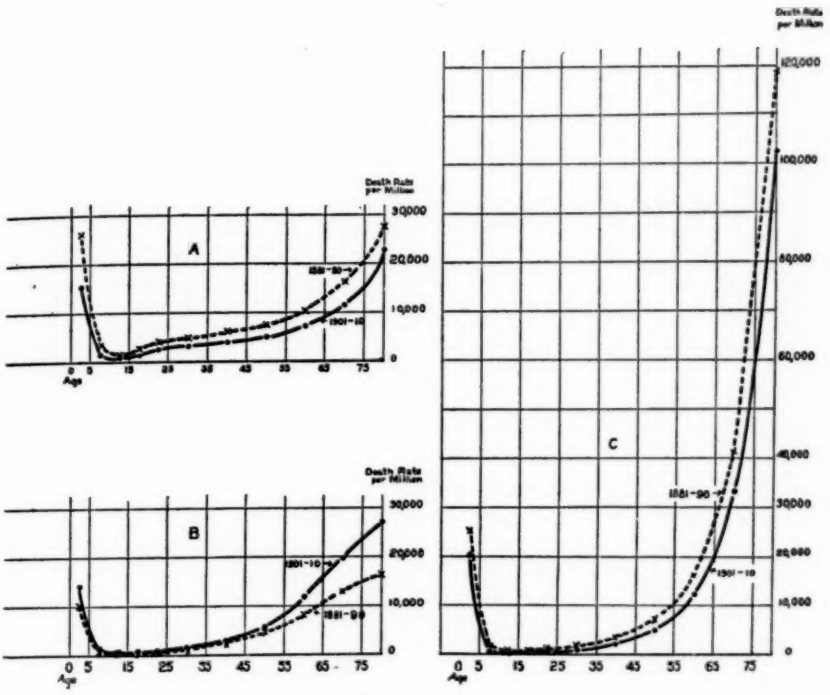


FIGURE VII. DEATH RATE PER MILLION MALES AT DIFFERENT AGE PERIODS IN 1881–90 AND 1901–10.

A. Group of diseases with decreasing death rate.*
B. Group of diseases with increasing death rate.*
C. All other causes of death.

that the balance is on the side of gain of life. For England we must decide between three possibilities: (1) increased mortality from certain diseases in middle and advanced life has been more than counterbalanced by decreased mortality from other diseases; (2) there has been merely transference of entries owing to gradually increasing accuracy of certification of deaths; or (3) these two factors are represented in the results to an uncertain extent.

For the registration area of the United States the situation is not so clear. As in England, there has been great increase in the registered mortality from certain diseases during middle and late life. Thus, Dr. Frederick L. Hoffman in a paper on "The Mortality from Degenerative Diseases," while deprecating hasty conclusions from imperfect data gives tables from which the following extracts are taken:

*See list on page 710.

TABLE X

PERCENTAGE INCREASE OR DECREASE IN THE DEATH RATE FROM ALL CAUSES AND FROM EACH OF THE FOLLOWING DISEASES BETWEEN 1900 AND 1915

	At ages				
	30–40	40–50	50–60	60–70	70–80
Apoplexy and cerebral softening	+29.4	+11.1	+18.8	+33.3	+39.7
Heart and arterial diseases	+6.3	+19.8	+30.0	+56.3	+81.7
Kidney diseases	+21.0	+1.8	+22.2	+43.0	+64.1
All causes	−22.6	−8.5	+0.9	+5.8	+4.2

The total death rate for all causes at ages over 45 has also increased among males in the registration area, though to a relatively small extent. Furthermore, the prospects of survival to old age in the registration area are on the average equal to those of the English population; and it is likely that the increased death rate from all causes and from these "degenerative diseases" is due to the introduction of foreign populations, who have lived in youth, and possibly after arrival in this country, an arduous life of overstrain under circumstances of lower civilization with excess of infectious diseases.

He would be foolish who, in view of the foregoing broad facts for the average population, committed himself to a pessimistic view as to middle and late middle life in the future either for the English or the American white population. A more favorable view, however, is not incompatible with the existence of circumstances favoring or even producing premature decay in special groups. But these circumstances do not exist or are more than counterbalanced for the mass of our populations, which chiefly determines the trend of our national statistics; and it is all the more satisfactory—especially from a public health standpoint—that this should be so, in view of the increasing trend toward city life.

POSSIBILITIES OF DIMINISHING DISEASES AT AGES 40 TO 70

There is ample evidence to show that apparently irrespective of modes of life, longevity is the rule in certain families, while in others, equally apart from perceptible differences in mode of life, the arteries fail prematurely. It would appear that there is no condition in which the influence of heredity is more marked than in the capacity to attain old age.

Apart from this unequally distributed inherent capacity for old age, we must, if asked to state in a single formula the most serious impediment to the attainment of old age, agree that *injury due to infections is the chief cause of mortality.*

Next to infections as a cause of premature death comes malignant disease, the most pitiless of enemies, depriving us by cruel steps of those whose ripe judgment and mature knowledge make them almost irreplaceable. It is between the ages of 40 and 60 that men begin to repay the community for the varied expenses hitherto incurred on their account. Prior to this period the balance is often on the wrong side, and it is at these ages that cancer chiefly claims its victims.

In men in England at ages 45–55, one death out of 10.5 total deaths from all causes, at ages 55–65, one death out of 8.1, at ages 65–75, one death out of 9.6, and at ages over 75 one death out of 19.3 total deaths was due, in 1901–10, to this cause. And it still remains true that although in every civilized country earnest investigators are searching for a line of action which may be hopeful if not certainly successful, at present we can only point to the avoidance of protracted local irritations, and to the early diagnosis and treatment of the disease, as means of prevention of death in a certain proportion of cases.

We must confess our almost equally great inability with respect to catarrhal infections, whether ordinary catarrhs or their more serious congeners, bronchitis and pneumonia. Apart from avenues of hope with regard to pneumococcic infections, there is little prospect of early conquest over these diseases.

Pandemic influenza, with streptococcic secondary infection, has recently proved to be a more serious cause of death than a world war, and no action for its abatement, on a large scale, has been practicable.

Tuberculosis, although capable of being reduced to a shadow of its present importance, were we prepared to invest the necessary money and energy in continuous and complete action against it, still stalks the earth and cuts off a large share of our population, especially at ages when persons are just beginning to repay their communal indebtedness. Tuberculosis is also, much more than is recognized, a common cause of death at ages over 50, either as a chronic disease, often masquerading as senile bronchitis, or as an acute complication of other diseases. As tuberculosis becomes relatively less serious, cancer more than takes its place as a cause of premature mortality.

Diseases of the heart and blood vessels form a complex group and have diverse causation. Of the evil influence of excessive muscular work, of over-feeding, alcoholism, or excessive smoking, or other unhygenic personal habits in securing prematurely senile arteries and heart, I will assume that there is no doubt; but it is practically certain that such factors are of relatively small importance as compared with the havoc played by the multiple infections to which we are subjected. The chief enemies which prevent arteries and heart from fulfilling

their duties to a robust old age are the specific infections of rheumatic fever, of syphilis, the pneumococci, and various streptococci from focal or other infections and still oftener secondary to an attack of an acute specific infectious disease.

We are now thoroughly advised of the magnitude of the mischief done by syphilis. Cerebral hemorrhage before the age of 40 or 45 may be assumed nearly always to owe its origin to this disease; and we know that more than one-tenth of the inmates of our lunatic asylums are there owing to syphilis, and die a premature death because of this infection. Will the community have the courage and wisdom to adopt the medical, police, social, and moral measures required to reduce it to insignificance?

It must be confessed that but little more appears to be known concerning rheumatic fever than when, in the Milroy Lectures for 1895,* I showed by elaborate mortality and sickness statistics derived from the general mortality experience of different European countries, from the general notification experience of Scandinavian countries, and from the experience of large general hospitals in England and other countries, that rheumatic fever is an epidemic disease of which widespread epidemics occur at intervals of a few years, though in the intervals it is never entirely absent from most communities. I drew attention to pandemics of rheumatic fever, particularly those of 1868, of 1874–75, and of 1884. I also showed that in England the epidemic prevalence of rheumatic fever has always occurred in years of exceptional scarcity of rainfall.

The causation of renal diseases or of arteriorsclerosis, apart from the influence of acute infectious diseases, is still obscure. We can, however, assert with a high degree of probability that if rheumatic fever could be avoided, if syphilis could be eliminated, and if the acute and chronic infectious diseases of childhood and youth and early manhood could be reduced to a shadow of their present dimensions, there would result an immense leap forward in the standard of health of the general community, and in the number of persons attaining a stalwart and healthy old age.

The chief and most promising line of attack on the disabling diseases of middle life and even of higher ages consists in the adoption of all known preventive and curative medical measures in childhood and in youth, adolescence and early manhood. It is chiefly in these years that the bill is incurred which has to be paid two, three, or four decades later. This does not imply, of course, that efforts made to anticipate and retard the onset of illness in older persons are fruitless. They are very desirable.

* *Lancet*, May 9 and 16, 1895.

We have in fact a quadruple line of action open to us in securing a healthy life of physiologically normal duration:

1. To pursue the present lines of public health activities with complete efficiency, and thus reduce the prevalence of the infectious diseases (chronic and acute) which, though now controllable, are not controlled.

2. To undertake every additional line of public and private control of disease in the first twenty-five years of life which our more recent knowledge of preventive medicine shows to be practicable.

3. To adopt the same measures, as far as they are applicable, to ages over 25.

4. To follow every line of investigation which may enable us to secure further control over disease.

There can be no doubt that although every line of action indicated above must be followed to ensure success, the greatest and earliest results will be secured by a great increase of activities under the first two of these headings.

WITH INCREASE OF AVERAGE LENGTH OF LIFE HAS THERE BEEN IMPROVED HEALTH?

The facts already stated prove not only that out of a given number born, a larger number than formerly survive to the more useful years of life and to old age, but also that out of equal numbers taken at age 25 or even at age 40, the prospects of survival to old age have improved. Any doubt on this point, for the white population of the United States, results probably from the vast introduction of a foreign population belonging in large measure to a lower order of sanitary and social organization.

But these facts are not incompatible with the possibility that increased survival means a large amount of invalidism and inferior health in the population. Are we in the fullest sense living longer, or are we merely longer in dying? This problem cannot be discussed adequately in the present paper. I may be permitted to quote the following remarks written by me in 1893. At that time the death rate had increased in England at the higher ages, as it has recently done in the United States. But the general comments are relevant still.*

"1. A favorite explanation of the diminished expectation of life in adult years is that owing to the saving of life in the earlier years of life—a saving which has been especially in zymotic diseases and phthisis and other tubercular diseases—there has been a larger number of

* See Brighton Life Table, 1893, or *Elements of Vital Statistics*, p. 316.

weakly survivors, who would under the former régime have been carried off by these diseases. In other words, the operation of the law of the survival of the fittest has been impeded, with results unfavorable to the health and vigor of adult life. This argument assumes that weakly children are more prone to attack by infectious diseases than robust children, an assumption which experience does not confirm. These diseases appear to attack the majority of children, weakly or robust, who are exposed to their infection. It might be reasonably expected, therefore, that with a decrease in the total deaths from infectious diseases, there would have been at least a corresponding decrease in the number of those who are left maimed by an attack of one of these diseases to survive to adult life. I personally think that the weeding out of weakly lives, caused by the greater mortality among weakly children suffering from an infectious disease, is almost entirely counterbalanced by the greater number of children made weakly in former times by non-fatal attacks of an infectious disease.

The case for deterioration of the race by survival of patients who would formerly have died in early life from phthisis and other tubercular diseases, appears to be a stronger one. It is probable that a larger proportion of phthisical patients are cured than formerly. It is probable also that many more children with a strong tendency to phthisis or even suffering from its early symptoms are prevented by the improved medical treatment and the improved social conditions of recent years from developing the disease. These now may survive to adult life and become the parents of children with a strong tubercular tendency.

Such a fact need not, however, cause any serious apprehension for two reasons. In the first place, hereditary tendencies to phthisis act only under favorable predisposing conditions, such as damp and overcrowded houses, sedentary occupation in a cramped position, etc.; and in presence of the active exciting agent, the specific bacillus to which phthisis and other tubercular diseases are due. The exciting cause of tuberculosis is the introduction *ab extra* of the specific infection by inhalation or by means of food.

In the second place, assuming that more phthisical patients survive than formerly, is it not equally true that fewer persons *become* phthisical than formerly? With a diminution of the active cases of phthisis, the number of centers for phthisical sputum, the chief cause of subsequent infection, must have diminished to a corresponding extent. Of the fact that the predisposing causes of phthisis—damp and overcrowded houses, ill-ventilated workshops, etc.—are steadily diminishing, there is evidence on every hand. It is, therefore, reasonable to suppose that much at least of the deteriorating effect of survival of tubercular persons is counterbalanced by the large number of persons who are *prevented by improved sanitary and social conditions from becoming tubercular.*

It is premature at present to attempt by statistical means to determine how far the counteracting influences which are at work balance each other, or failing a balance, on which side is the preponderating effect.

2. The increased stress of modern life is supposed by many to explain the increased death rate among adults. It is doubtful if such increased strain exists in the community as a whole. Each adult as he becomes year by year more deeply involved in the battle of life, comes to the conclusion that the general strain of life in the community is increasing, forgetting that the same causes operated as life advanced in previous generations. There is reason for thinking with Dr. Pye-Smith that much of the evil ascribed to "over-pressure" is really due to over-feeding and drinking.

Assuming, however, that over-pressure exists in certain stations of life, *e. g.*, among city merchants, medical men, etc., it cannot be said to exist generally among professional men. Clergymen, lawyers, and civil-servants are as classes long-lived.

Even assuming that over-pressure exists throughout the whole of the professional and mercantile classes, these do not form the mass of the community. *The majority of the population of England and Wales belongs to the wage-earning classes*, and the conditions of these classes will therefore necessarily have the greatest influence on the total result.

Those conditions, as we know, have greatly improved.

I see no reason for altering the view stated in the preceding extract, except that I should now attach little if any importance to hereditary predisposition in tuberculosis, and should state my conclusions with less hesitation. Under the circumstances of modern civilization the assumption that natural selection can act as under savage conditions is completely unwarranted. Civilized life, leading to progressive improvement of environment and of personal habits, submerges any possible influence of natural selection in removing those unfit for survival, substituting for it a process of steady uplifting in fitness of the general population.

This is well illustrated in the following curves which show that infants who have escaped the assumed selective influence of a high infant mortality in infancy, continue to survive all through life in larger numbers than infants among whom excessive infant mortality has prevailed.

The recent recruiting figures have been adduced as evidence of widespread physical deterioration, and they doubtless show that both in this country and in the United Kingdom a large proportion of recruits suffered from physical defect or disease, rendering them unfit for military service. I have no wish to minimize the importance of these figures. That they indicate any deterioration in physique in the population, historically, is not proved, and is highly improbable.

In 1844 over half the recruits in Leeds were rejected; and in Birmingham and the surrounding towns in 1852 only one-third of the

	Mean Population, 1891–1900.	Mean Infant Mortality per 1,000 Births.
England and Wales	30,643,479	.. 157
Selected Healthy Districts ..	4,477,485	.. 109

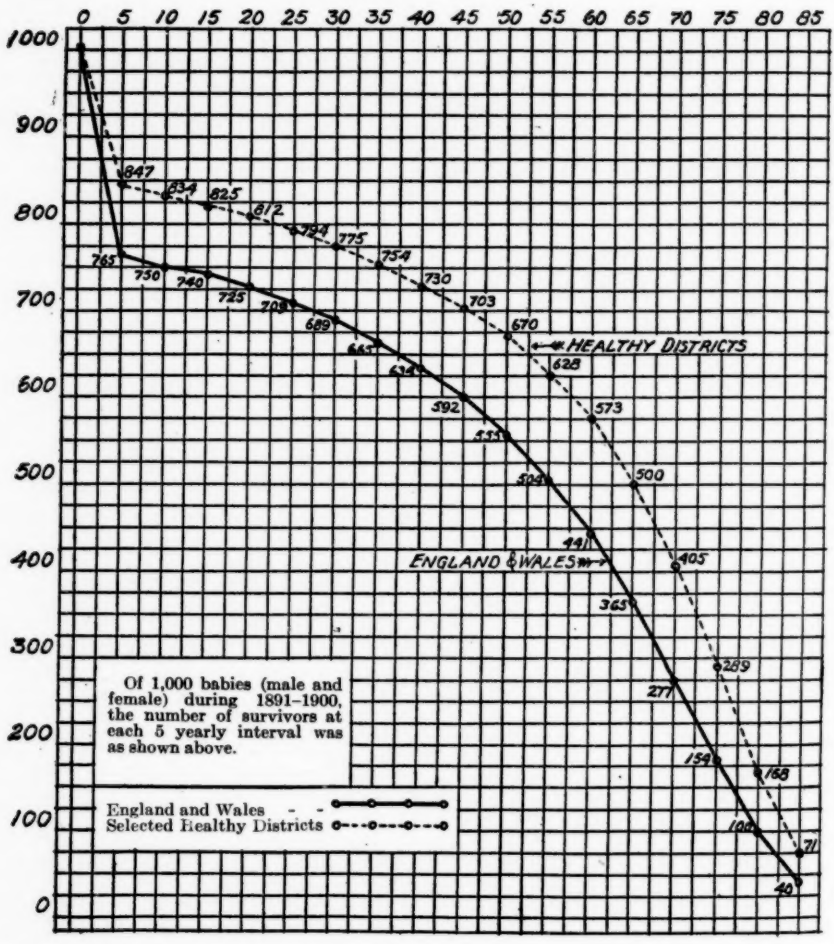


FIGURE VIII. Showing the number of survivors at each successive year of life out of 1,000 infants born in England and Wales and in selected healthy districts in 1881–90.

This table is to be read as shown in the following example:—At age 20 the number of survivors was 812 in healthy districts, 726 in the country as a whole; at age 60 was 573 in healthy districts, 441 in the country as a whole; and so on.

men who enlisted were approved. We have always thought we were a decadent race, and so have other nations before us. The ancient Greeks thought themselves to be degenerate, and the Spartans adopted the ill-considered action of exposing weakly infants to avoid this result. Sophocles is quoted by Dean Inge as stating that the best fate of man is "not to be born, or being born to die." Shakespeare refers to this "waning age" and has other similar passages. In 1721 Bishop Berkeley wrote an "essay towards preventing the ruin of Great Britain"; and you will remember Wordsworth's sonnet in which the following passage appears:

> "Milton, thou should'st be living at this hour,
> England hath need of thee: She is a fen
> Of stagnant waters";

though he adds later:

> "It is not to be thought of that the Flood of British freedom
> * * *
> That this most famous stream in bogs and sands
> Should perish; and to evil and to good
> Be lost for ever."

Every Tory Squire still believes that England is going to the dogs; but I prefer the sanguine view expressed by Tennyson as embodying the real outlook of both England and America:

> "This fine old world of ours is but a child,
> Yet in the go-car."

THE MATHEMATICAL REPRESENTATION OF FREQUENCY DISTRIBUTIONS

By Harry C. Carver, *University of Michigan*

Section I. distributions of graduated variates
Section II. distributions of integral variates
Section III. difference equation graduation*
Section IV. application of the hypergeometric series*

$$(A) \quad u_x = {}_{pn}C_{r-x} \; {}_{qu}C_x$$
$$(B) \quad u_x = {}_{pn}H_{r-x} \; {}_{qu}H_x$$

As an introduction to this paper on mathematical statistics the writer cannot refrain from quoting the first paragraph from the introduction of an extremely sound treatise on non-mathematical statistics, Zizek's *Statistical Averages*, a translation by Persons.†

"In nearly all branches of statistical investigation averages or means are very often used for various purposes of the greatest significance. Averages are, therefore, undoubtedly to be reckoned among the most important aids in statistical method. Even Guerry, the founder of moral statistics, indicated the extraordinary importance of averages in the following definition of statistics: 'The science of statistics consists essentially in the methodological enumeration of variable elements, whose mean it determines.' Edgeworth defines statistics as 'the science of those means which are presented by social phenomena'; and similarly Bowley says, 'Statistics may rightly be called the science of averages.' The application of averages has, as is well known, given rise to controversies of various kinds, which fill a considerable part of statistical literature. Not infrequently the incorrect use of averages has also led to erroneous conclusions and to contradictions which have shaken our confidence in statistics. Averages, indeed, are only applicable under strictly defined conditions, and conclusions based on averages are likewise permissible only within well defined limits. It is the task of statistical science to investigate the application and use of averages from the general methodological standpoint, and to determine the part which averages should play in statistical method."

In astronomy and in other fields in which precise measurements are necessary, the average of a series of independent measurements of equal weight on an unknown quantity is assumed to give the best approximation to, *i. e.*, the most probable value of, the magnitude in question. But the computation of the mean, instead of permitting a final answer to the problem, only gives rise to other questions of a more involved character. Thus, we may require the "probable error"

* Sections III and IV of this paper will appear in the September issue of the Quarterly Publications.
† New York: Henry Holt and Company, 1913.

of the mean in order to estimate the extent to which we may rely on this average, or if this mean is to be combined with other means, it is necessary to know the approximate "weight" of each in order to insure a proper combination of observations.

In statistics it is likewise of the utmost importance that we be able to measure the dependability and the significance of averages. Thus, it may be quite necessary that we know that the average or normal weight of a male infant sixteen months old is 23.25 pounds, or that the average weight of a man who is thirty-seven years old and six feet tall is 179 pounds; but it is likewise important that we know to what extent these averages may be relied upon and be able to state to what extent a sixteen months old infant weighing 19 pounds or a thirty-seven year old male six feet in height and weighing 200 pounds, is abnormal.

Again, if from past experience we observe that certain abnormal conditions of rainfall and temperature seem to be associated with abnormal crops, we immediately feel that it would be desirable to know to just what extent these phenomena are on the average associated, and to be able to determine, if possible, a functional relation between cause and effect. Assuming that a mathematical representation of the law of cause and effect is obtainable, we must hesitate to place any confidence on any dependent prediction until we have investigated the dependability of the mathematical predictions.

It is quite clear that the dependability of a mean must be a function of both the number of observations used in computing the mean and the magnitudes of the variations about the mean. It follows,therefore, that a critical investigation of the distribution of magnitude-determinations about their mean is essential.

The fundamental problem of mathematical statistics is to develop an analytical expression that is capable of representing any distribution of magnitude-determinations, *i. e., variates.* The Gaussian or Normal Curve of Error, Pearson's Generalized Probability Curves, and Charlier's Type A and Type B curves are all the results of investigations of this fundamental problem.

In this paper we shall first investigate the general problem of distribution from the point of view of a general mathematical representation rather than from the point of view of the theory of probability. The writer believes it possible to justify this deviation from the usual method of presentation.

Section I

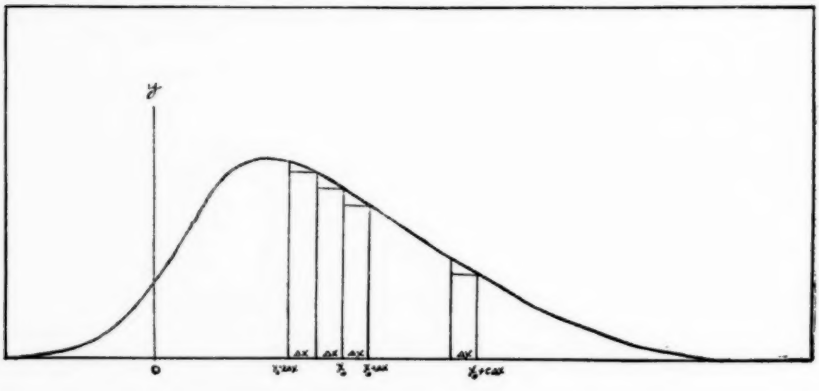
DISTRIBUTIONS OF GRADUATED VARIATES

Definition: *Graduated variates* are expressions of observed magnitudes which, from their nature, do not exist as integers but which may differ by infinitesimals.

We assume that variations from the mean follow some unknown law and that the observed class frequencies are approximations to corresponding areas under a theoretical *frequency function*, which affords a mathematical representation of the unknown law of distribution.

Let $f(x)$ be any frequency function which vanishes at $x=-l_1$ and $x=+l_2$, where the origin is taken at any point within the range. The limits may be either both finite, both infinite, or one finite and the other infinite.

Let the range $(-l_1, l_2)$ be divided into segments of equal length, Δx, and let ordinates be erected at each of the points of division. Now let rectangles be inscribed on these segments with altitudes equal to their respective right-hand boundaries.



Choosing one of these points of division as origin, the area of the rectangle whose right-hand boundary is at x_0 is

$$\Delta x \cdot f(x_0)$$

and that of any other rectangle whose right-hand boundary is $c \cdot \Delta x$ distant is

$$\Delta x \cdot f(x_0+c\cdot\Delta x).$$

Let us now set up the function

$$\sum_{c=-\frac{l_1+x_0}{\Delta x}}^{\frac{l_2-x_0}{\Delta x}} \Delta x \cdot f(x_0+c\cdot\Delta x)e^{(x_0+c\,\cdot\,\Delta x)\omega i}$$

where $i=\sqrt{-1}$ and ω is a parameter, and observe that if we multiply each term of this summation by

$$e^{-x_0\omega i}$$

and integrate the resulting series term by term with respect to ω we obtain the following identity

$$\int_{-\pi/\Delta x}^{\pi/\Delta x} e^{-x_0\omega i}\left\{\sum_{c=-\frac{l_1+x_0}{\Delta x}}^{\frac{l_2-x_0}{\Delta x}} \Delta x \cdot f(x_0+c\cdot\Delta x)e^{(x_0+c\,\cdot\,\Delta x)\omega i}\right\} d\omega = 2\pi f(x_0).$$

This follows immediately from the fact that the general term of the summation after multiplication by

$$e^{-x_0\omega i}$$

and subsequent integration yields, for $c\neq 0$

$$\Delta x \cdot f(x_0+c\cdot\Delta x)\int_{-\pi/\Delta x}^{\pi/\Delta x}(\cos c\cdot\Delta x\,\cdot\,\omega+i\sin c\cdot\Delta x\cdot\omega)d\omega=0,$$

since c is necessarily an integer.

For $c\equiv 0$ we have

$$\Delta x \cdot f(x_0)\int_{-\pi/\Delta x}^{\pi/\Delta x} d\omega = 2\pi f(x_0).$$

We have therefore

$$(1)\quad f(x_0)=\frac{1}{2\pi}\int_{-\pi/\Delta x}^{\pi/\Delta x} e^{-x_0\omega i}\left\{\sum_{c=-\frac{l_1+x_0}{\Delta x}}^{\frac{l_2-x_0}{\Delta x}} \Delta x \cdot f(x_0+c\cdot\Delta x)e^{(x_0+c\,\cdot\,\Delta x)\omega i}\right\} d\omega.$$

Now permitting Δx to approach zero we have

$$(2)\qquad f(x_0)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-x_0\omega i}\left\{\int_{-l_1}^{l_2} f(x)e^{x\omega i}dx\right\} d\omega.$$

Formula (2) is merely a form of Fourier's Integral.

If we now place

$$(3)\qquad \int_{-l_1}^{l_2} f(x)e^{x\omega i}dx = e^{c_0+c_1\omega i-c_2\frac{\omega^2}{2}}\left\{1+c_3\frac{\omega^3 i^3}{\underline{|3}}+c_4\frac{\omega^4 i^4}{\underline{|4}}+\ \cdots\right\},$$

the coefficients, c_n, may be evaluated by expanding

$$\int_{-l_1}^{l_2} f(x)e^{-c_0+\omega i(x-c_1)+c_2\frac{\omega^2}{2}}\,dx$$

in a power series of ω and imposing the condition that the coefficients of ω^0, ω^1 and ω^2 of the resulting series must equal 1, 0, and 0 respectively.

For values of c_n, $(n\geqq 3)$

$$c_n i^n = e^{-c_0}D_\omega^{(n)}\int_{-l_1}^{l_2} f(x)e^{\omega i(x-c_1)+c_2\frac{\omega^2}{2}}\,dx\Big]_{\omega=0},$$

and applying the operator formula

$$\phi(D)e^{ax}V = e^{ax}\phi(D+a)V$$

we may write

$$(4)\qquad c_n i^n = e^{-c_0}\int_{-l_1}^{l_2} f(x)e^{\omega i(x-c_1)}\left\{(D_\omega+i\sqrt{x-c_1})^n e^{c_2\frac{\omega^2}{2}}\right\}_{\omega=0}dx\Big]$$

$$= i^n e^{-c_0}\int_{-l_1}^{l_2}\left\{(x-c_1)^n-\frac{n^{(2)}}{2}c_2(x-c_1)^{n-2}+\frac{n^{(4)}}{2^2\underline{|2}}c_2^2(x-c_1)^{n-4}\right.$$

$$\left.+\ \cdots\right\}f(x)dx,$$

where

$$n^{(r)} = n(n-1)(n-2)\ \cdots\ (n-r+1).$$

Imposing the condition that (4) must equal 1, 0, and 0 for $n=0$, 1, and 2 respectively, we have

$$e^{-c_0} = \frac{1}{\int_{-l_1}^{l_2} f(x)dx},$$

$$c_1 = \frac{\int_{-l_1}^{l_2} x f(x)dx}{\int_{-l_1}^{l_2} f(x)dx},$$

$$c_2 = \frac{\int_{-l_1}^{l_2} (x-c_1)^2 f(x)dx}{\int_{-l_1}^{l_2} f(x)dx}.$$

We see, then, that c_1 is the distance from the arbitrary origin to the mean and that c_2 is therefore the second moment of unit frequency about the mean, or in other words is the square of the standard deviation.

Representing, as is customary, by

b, the distance from the arbitrary origin to the mean,

σ, the standard deviation,

N, the total frequency of the distribution,

ν'_n, the n^{th} moment of unit frequency about the arbitrary origin,

$$\text{i.e. } \nu'_n = \frac{\int_{-l_1}^{l_2} x^n f(x)dx}{\int_{-l_1}^{l_2} f(x)dx},$$

ν_n, the n^{th} moment of unit frequency about the mean

$$i.e. \ \nu_n = \frac{\int_{-l_1}^{l_2} (x-c_1)^n f(x)dx}{\int_{-l_1}^{l_2} f(x)dx}$$

we obtain,

$$(5) \qquad \nu_n = \nu'_n - {}_nC_1 b\nu'_{n-1} + {}_nC_2 b^2\nu'_{n-2} - \cdots$$

$$c_n = \nu_n - \frac{n^{(2)}}{2}\sigma^2\nu_{n-2} + \frac{n^{(4)}}{2^2\underline{|2}}\sigma^4\nu_{n-4} - \frac{n^{(6)}}{2^3\underline{|3}}\sigma^6\nu_{n-6} + \cdots.$$

Referring to equations (2) and (3) we see that

$$f(x_0)=\frac{N}{2\pi}\int_{-\infty}^{\infty}e^{-(x_0-b)\omega i-\frac{\sigma^2\omega^2}{2}}\left\{1+c_3\frac{\omega^3 i^3}{\underline{|3}}+c_4\frac{\omega^4 i^4}{\underline{|4}}+\cdots\right\}d\omega.$$

If we now set

$$\phi_{x_0}=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-(x_0-b)\omega i-\frac{\sigma^2\omega^2}{2}}d\omega$$

and note that

$$\phi_{(x_0)}{}^{(n)}=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-(x_0-b)\omega i-\frac{\sigma^2\omega^2}{2}}(-\omega i)^n d\omega$$

we have finally, dropping the subscript on x, that any ordinate

$$(6)\qquad f(x)=N\left\{\phi_x-\frac{c_3}{\underline{|3}}\phi_x^{(3)}+\frac{c_4}{\underline{|4}}\phi_x^{(4)}-\frac{c_5}{\underline{|5}}\phi_x^{(5)}+\cdots\right\},$$

where from equation (5)

$$(7)\qquad\begin{cases}c_3=\nu_3\\ c_4=\nu_4-3\sigma^4\\ c_5=\nu_5-10\sigma^2\nu_3\\ c_6=\nu_6-15\sigma^2\nu_4+30\sigma^6\\ c_7=\nu_7-21\sigma^2\nu_5+105\sigma^4\nu_3\\ c_8=\nu_8-28\sigma^2\nu_6+210\sigma^4\nu_4-315\sigma^8\\ \text{etc.}\end{cases}$$

From the well-known definite integral

$$\int_{-\infty}^{\infty}e^{-a^2x^2}\cos bx\,dx=\frac{\sqrt{\pi}}{a}e^{-\frac{b^2}{4a^2}}$$

we see that

$$(8)\qquad \phi_x=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-b)^2}{2\sigma^2}}$$

Since the coefficients, c_n, are functions of the unknown law of distribution it would appear at first thought that we are now as far from having obtained a workable analytical expression for any distribution of graduate variates as we were at the start.

But although fluctuations in the values of individual class frequencies may arise as the result of "random sampling," we assume that the values of ν_n as computed from any observed frequency distribution

will approximately equal the corresponding values which are functions of the *unknown* law of distribution. Certain theorems involving the theory of "probable error" enable us to ascertain the degree of approximation involved in this assumption.

It follows, then, that if equation (6) is so rapidly convergent that terms after the third or fourth may be neglected, it affords us a simple representation of any distribution.

The rapidity with which this series converges, however, depends upon the extent to which the generating function, ϕ_x, is a fair approximation of the unknown law of distribution. In general, if the distribution be approximately symmetrical, equation (6) is very rapidly convergent. If, however, the distribution be bimodal or extremely asymmetrical, series (6) as a function of the symmetrical generating function, ϕ_x, is for practical purposes of no value because

(a) the number of required terms in (6) would necessitate an impracticable amount of labor; and

(b) the probable errors of c_n for values of n greater than 5 or 6 are generally so large that the assumption that, for these, we may substitute the moments computed from the observed data for the theoretical moments, is open to serious criticism.

It should be understood, however, that theoretically equation (6) is capable of representing any frequency function associated with graduated variates, but that convergence means one thing and rapid convergence quite another.

In case the distribution is skew we may proceed as follows:

Let θ_x be any arbitrarily chosen function which may be used as a statistical generating function and which, being a frequency function, may be expressed by means of equation (6) in the series

$$\theta_x = M\left\{\phi_x - \frac{d_3}{\lfloor\underline{3}}\phi_x^{(3)} + \frac{d_4}{\lfloor\underline{4}}\phi_x^{(4)} - \frac{d^5}{\lfloor\underline{5}}\phi^5{}_x + \cdots\right\}, \tag{9}$$

where d_n is a function of the moments of θ_x.

Eliminating ϕ_x between (6) and (9) we obtain

$$f(x) = \frac{N}{M}\left\{\theta_x - \frac{A_3}{\lfloor\underline{3}}\theta_x^{(3)} + \frac{A_4}{\lfloor\underline{4}}\theta_x^{(4)} - \frac{A_5}{\lfloor\underline{5}}\theta_x^{(5)} + \cdots\right\} \tag{10}$$

where

$$\begin{aligned} A_3 &= c_3 - d_3 \\ A_4 &= c_4 - d_4 \\ A_5 &= c_5 - d_5 \\ A_6 &= (c_6 - d_6) - 20d_3(c_3 - d_3) \\ &\cdots\cdots \end{aligned}$$

The area under $f(x)$ between any two limits may now be computed since

$$(11) \qquad \int_a^b f(x)dx = \frac{N}{M}\left[\int \theta_x dx - \frac{A_3}{\underline{3}}\theta_x^{(2)} + \frac{A_4}{\underline{4}}\theta_x^{(3)} - \ \ldots\right]_a^b.$$

Inasmuch as tables for both the definite integral from 0 to x and $D^{(n)}(n=1, 2, 3, 4, 5, 6)$ have been computed for

$$\phi_x = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-b)^2}{2\sigma^2}}$$

it follows that in problems for which ϕ_x may be applicable, we are in a position to proceed with numerical computations.

It is highly desirable, however, that investigations be made in the endeavor to establish a reference list of generating functions possessing various desired properties. If the indefinite integral of any of these generating functions could readily be computed, such an investigation would be of the greatest value.

Again, since in the making of computations it would be much easier to obtain the numerical value of the n^{th} *difference* of θ_x than for the n^{th} *derivative*, it will be found of frequent advantage to utilize the following relation

$$D^n = \left(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \ \ldots\right)^n$$

and write (10) and (11) as

$$(12) \qquad f(x) = \frac{N}{M}\left\{\theta_x - \frac{B_3}{\underline{3}}\Delta^3\theta_x + \frac{B_4}{\underline{4}}\Delta^4\theta_x - \ \ldots\right\} \text{ and}$$

$$(13) \qquad \int_a^b f(x)dx = \frac{N}{M}\left[\int \theta_x dx - \frac{B_3'}{\underline{3}}\Delta^2\theta_x + \frac{B_4'}{\underline{4}}\Delta^3\theta_x - \ \ldots\right]_a^b \text{ respectively,}$$

where

$$B_3 = A_3 \qquad B'_3 = A_3$$

$$B_4 = A_4 + 6A_3 \qquad B'_4 = A_4 + 4A_3$$

$$B_5 = A_5 + 10A_4 + 35A_3 \qquad B'_5 = A_5 + \frac{15}{2}A_4 + \frac{55}{3}A_3$$

$$B_6 = A_6 + 15A_5 + 85A_4 + 225A_3 \qquad B'_6 = A_6 + 12A_5 + \frac{105}{2}A_4 + 100A_3$$

. . . .

In order that formulas 12 and 13 may be useful, it is necessary that the class interval be taken as the unit of x for θ_x and for all computed moments.

Section II

DISTRIBUTIONS OF INTEGRAL VARIATES

Definition: *Integral variates* are expressions of observed magnitudes which, from their nature, exist only as integers. For example: number of petals of a flower, number of persons in a family, etc.

We assume that variations from the mean follow some unknown law and that the observed class frequencies are approximations to corresponding *ordinates* of the theoretical frequency function which affords a mathematical representation of the unknown law of distribution.

We may readily write down an expression from which we may develop the general law of distribution for integral variates by placing c equal to unity in formula (1) of the preceding section. We have, then,

$$f(x_0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-x_0\omega i}\left\{\sum_{x=-l_1}^{l_2} f(x)e^{x\omega i}\right\}d\omega. \tag{14}$$

Placing now

$$\sum_{x=-l_1}^{l_2} f(x)e^{x\omega i} = e^{c_0+c_1\omega i-c_2\frac{\omega^2}{2}}\left\{1+c_3\frac{\omega^3 i^3}{\lfloor 3}+c_4\frac{\omega^4 i^4}{\lfloor 4}+\ \ldots\right\}$$

and proceeding exactly as before, we obtain, corresponding to (6),

$$f(x) = N\left\{\phi_x - \frac{c_3}{\lfloor 3}\phi_x^{(3)} + \frac{c_4}{\lfloor 4}\phi_x^{(4)} - \ \ldots\right\} \tag{15}$$

where now the values of c_n are the same as those given in (7) if we now define

$$\nu_n = \frac{\Sigma(x-c_1)^n f(x)}{\Sigma f(x)}$$

and

$$\phi_x = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-(x_0-b)\omega i-\frac{\sigma^2\omega^2}{2}}d\omega. \tag{16}$$

This ϕ_x may be eliminated as before, producing expansions corresponding to (10) and (12).

Discussion of Sections I and II

In *Über das Fehlergesetz* and *Die zweite Form des Fehlergesetzes*, Charlier has shown that on the hypothesis of elementary errors any law of error may assume one of the two forms.

Type A

$$\phi_x+A_3\phi_x^{(3)}+A_4\phi_x^{(4)}+\ \cdot\ \cdot\ \cdot$$

Type B

$$\psi_{(x)}+B_1\Delta\psi_{(x)}+B_2\Delta^2\psi_{(x)}+\ \cdot\ \cdot\ \cdot$$

where

$$\phi_x=\frac{1}{2\pi}\int_{-\pi}^{\pi}e^{-(x_0-b)\omega i-\frac{\sigma^2\omega_2}{2}}d\omega,$$

and

$$\psi_{(x)}=\frac{e^{-\lambda}}{\pi}\int_0^{-\pi}e^{x\cos\omega}\cos\ (\lambda\sin\omega-x\omega)d\omega$$

$$=\frac{e^{-\lambda}\lambda^x}{\lfloor x}$$

if x be an integer.

Owing to an oversight, π instead of ∞ appears as each limit of ϕ_x. The constants A_n and B_n are determined in *Über die Darstellung willkürlicher Funktionen* by applications of the method of moments, but appear to differ from the ones developed herein for values of n greater than (5) since by our general formula (5),

$$c_6=\nu_6-15\sigma^2\nu_4+30\sigma^6,$$

whereas Charlier gives

$$c_6=\nu_6-15\sigma^2\nu_4+15\sigma^6.$$

For values of n less than 6 the results agree.

As the result of the present investigation it would appear that there is no essential difference between Charlier's Type A and Type B curves, aside from the fact that both are merely special cases of formulas (10) and (12) when we define our generating functions as

$$\theta_x=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-b)^2}{2\sigma^2}}$$

and

$$\theta_x=\frac{e^{-\lambda}\lambda^x}{\lfloor x}\ \text{respectively.}$$

The fact that the second generating function may assume asymmetrical as well as nearly symmetrical forms renders the Type B curves more powerful than those of Type A.

It would appear, however, that for graduated variates the continuous limit of Poisson's Exponential Binomial Limit, namely, Pearson's Type III curve,

$$y = y_0 e^{-\gamma x}\left(1+\frac{x}{a}\right)^{\gamma a}$$

placing $\nu_2 = \nu_3$, might be more logically applied.

Again, if we do not impose on the above generating function the condition that $\nu_2 = \nu_3$, it will be noted that any law of distribution may be written,

$$f(x) = N\left\{\theta_x + \frac{A_4}{\underline{4}}\theta_x^{(4)} - \frac{A_5}{\underline{5}}\theta_x^{(5)} + \cdots\right\},$$

where θ_x is the above mentioned Pearson's Type III function. Again we might use

$$f(x) = N\left\{\theta_x + \frac{B_4}{\underline{4}}\Delta^4\theta_x - \frac{B_5}{\underline{5}}\Delta^5\theta_x + \cdots\right\}.$$

Ordinarily

$$f(x) = N\left\{\theta_x + \frac{B_4}{\underline{4}}\Delta^4\theta_x\right\}$$

will give practically the same results as any one of Pearson's Generalized Probability Curves, and will involve far less labor.

THE ORIGIN AND DEVELOPMENT OF MODERN TRADE STATISTICS

By Faith M. Williams, *Federal Reserve Board*

The compilation of foreign trade statistics is not a custom of recent origin. As M. Fernand Faure points out in his history of the "Development and Progress of Statistics in France,"* ordinances of the first quarter of the fourteenth century in France must have necessitated records of foreign trade. In 1305, Philip the Fair appointed a "Maitre des Ponts et Passages" whose duty it was to supervise, through the King's agents, all the goods entering and leaving French seaports; and an ordinance of 1324 imposed an export duty to be levied on the quantity, not on the value, of all goods leaving France.

Edward the Third seems to have been the originator of customs statistics in England. In 1374 we find that Geoffrey Chaucer (more famous as a poet than as a statistician) was appointed "Comptroller of the Customs, and Subsidy of Wools, Skins, and Tanned Hides in the Port of London, with the obligation to keep the records of his office with his own hand and to be continually present."† Few of these early records have been preserved, however. M. Chamberland, of the College of Chartres, published in 1892–3 in *La Revue de Geographie* a record of the commodities imported into France from Spain, Portugal, the Netherlands, Italy, the Levant, Germany, and England, which was drawn up somewhere between 1551 and 1556‡; but this document is almost unique. In general, little importance was attached to figures on foreign trade until the Mercantilist school appeared in the seventeenth century. The result of their convictions as to the necessity for a favorable balance of trade was a complicated network of commercial regulations and discriminating tariffs.§ The early Mercantilists evolved their theories and enacted laws enforcing them without any complete foreign trade statistics on which to base their reasoning, and by the end of the seventeenth century people began to see how dangerous their policies might become, particularly if they were not in accord with the real facts in the case.‖

* *The History of Statistics*, published for the American Statistical Association, pp. 232–233.

† *Life of Chaucer*, Globe Edition, p. xvii.

‡ E. Levasseur, *Histoire des Classes ouvrieres avant 1789*. Vol. II, pp. 49–50.

§ As in England the prohibition of the export of gold and the Navigation Acts of Cromwell, and in France the high tariffs of Colbert.

‖ See Sir Dudley North's *Discourses on Trade*, 1691, and Pierre Boisguilbert's *Detail de la France sous le regne present*, 1697.

England seems to have been the first country to authorize the preparation of complete tables of imports and exports. As early as 1626 Charles I appointed a Commission of Trade, (1) to advance home Commodities, and (2) to repress the ungainfull Importacion of forraigne Commodities to the end Trade maie bee ballanced . . . .* Among the commissioners named in the decree as "fitt for this Imployment" is Thomas Mun, the author of *England's Treasure by Foreign Trade*† which gives one of the clearest accounts ever written of the balance of trade theory. The Commission of Trade established by Charles I disappeared about 1640, but it was revived under Cromwell.‡ During the latter part of the seventeenth century England was flooded with essays on trade, which were filled with estimates of increase in trade, balance of trade, and revenue from trade, culled from the incomplete records of the time. Sir Josiah Child published a *New Discourse on Trade* in 1668 and revised it in 1690; Sir William Petty combined essays on the trade of England, France, and Holland in a volume called *Essays in Political Arithmetic;* Sir William Temple wrote *Observations upon the United Provinces of the Netherlands* in 1672, and an *Essay on the Trade of Ireland* in 1673; and lesser writers like Dudley Digges, George Malynes, Edward Misselden, and Samuel Fortrey published books on similar subjects from 1615 to 1673, all without either complete nor reliable figures on which to base their arguments. Few of these men wrote as if they felt the need of more adequate information.

However, Sir William Petty and his friend John Graunt,§ who had determined to express themselves only in terms of "number, weight and measure," were not without their influence. In 1695 the House of Lords directed the Commissioners of the Customs to draw out an "accompt of all the Exportations from London and the Out-ports to every distinct country; also of all the Importations to London and the Out-ports from every distinct country, for the years 1694, 1695, 1696 and so on downwards. Every commodity is under a separate Head and the draw-backs upon Re-exportation are taken notice of, and the value is set down where the duty on goods is ad valorem."‖ Charles Davenant, an admirer of Petty's methods (although he deplored Petty's lack of statistical material), was instrumental in bringing this order

* W. Cunningham, *The Growth of English Industry and Commerce in Modern Times.* Part I, p. 175. Note.

† Published in 1664, twenty-three years after Mun's death.

‡ Cunningham, p. 175. Note. This commission has been continued in England until the present time as the Board of Trade.

§ Author of "Observations" upon the London Bills of Mortality.

‖ Davenant, p. 32.

about. In his little book *Of the Use of Political Arithmetic* (London, 1698), he says:*

"There is hardly a Commerce, but the Dealers in it will affirm, we loose by all the rest; and yet 'tis evident that in time of Peace, the Kingdom gets by Trade in general.

"A true Account of the Ballance of Trade will set all this aright, and show what traffics are Hurtful and what are beneficial to the Nation; but probably the Ballance is in no way to be found out, but by Political Arithmetic. . . . He that would, therefore, compute with any good effect in matters relating to trade must contemplate the Wealth, Stock, Product, Consumption, Shipping, Exportations and Importations of his Country; and at the same time he must consider the State and Conditions of other places.

"In this Art, the most difficult Point is to find good materials and to have a Footing, probably sure, to fix our Reasonings upon . . . and in order to do this the House of Lords did in 1695 lay a most excellent foundation . . . and by this aid we hope to find out that Ballance of Trade which has been so often talked of."

It must not be supposed, however, that these complete records of trade, once begun, were published in detail. At stated intervals the total values of imports and exports were presented to Parliament, and on its request accounts of the trade in certain commodities—in corn, wine, or wool, or of the trade with certain countries—with France, the North American Colonies, or India were drawn off and inserted in the Parliamentary Papers.

It is now possible, therefore, to find an infinite variety of official foreign trade figures for the trade of the United Kingdom from 1694 on. However, no attempt was made to publish the complete details of that trade until the second quarter of the nineteenth century. Beginning in 1798 both the declared and the official values of the export trade had been recorded, and it soon began to be apparent that there were wide differences between the official and declared evaluations. A certain dissatisfaction with the methods used in compiling trade statistics grew up. In 1832 an attempt was made to present a complete picture of England's foreign trade from the material then available in the customs houses. In the Parliamentary Papers for that year there appeared† a detailed account of the quantity of the imports and the quantity and declared value of the exports of the United Kingdom by commodities and countries. The title of the document is as follows: "A Statement of the Imports and Exports of the United Kingdom for the year 1830, ended 5th January, 1831; distinguishing the Trade with each Foreign Country or British possession; and showing the principal Articles of Import and Export with the Quantities of such Articles, and in the case of British Merchandise, their declared

* Pp. 30–31.

† Parliamentary Papers, 1831–32. Vol. 34.

value, together with a Statement of the Quantities of Imported Articles Retained for Home Consumption." (Mr. Courtenay.*)

A more systematic presentation was desired, however, and in 1833, more than a century after complete records of foreign trade were begun, a special statistical division, under the charge of Mr. G. R. Porter,† was added to the Board of Trade. Within a short time Mr. Porter placed before Parliament an abstract of the annual trade of the United Kingdom from 1820 to 1831‡ which attempted to arrange the available facts with regard to this trade in an orderly and systematic fashion. The abstract contains the official values of the annual import, export, and re-export trade of the United Kingdom, the declared value of the export trade, and detailed figures with regard to the commerce in some 147 commodities. The details given as to the trade in each commodity vary greatly; in some cases there are only the quantities imported and exported each year; in the case of the grains the figures represent imports into *Great Britain*, including grain from Ireland; imports of cotton and wool and their manufactures are stated in great detail, distinguishing countries of origin and destination. In November, 1833, Mr. Porter published§ an account of the quantities of some 203 different commodities which entered into the foreign trade of the United Kingdom in 1831 and 1832, giving for some commodities countries of origin and destination. From that time on the communication to Parliament of detailed and systematically compiled statistics of imports and exports became an annual custom. In 1854, the method of publishing these statistics changed, and there appeared Volume I of the *Annual Statement of the Trade and Navigation of the United Kingdom*, in much the form in which it is now published.

France was not far behind the United Kingdom in beginning complete records of foreign trade. Arnould, in *La Balance du Commerce*,‖ says that in 1603 Sully used the registers of the five great custom houses to estimate the amount of revenue which would be received from an export tax of 15 sous per bale of merchandise exported. Colbert is supposed to have consulted these same records to get information as to importations, but neither Sully nor Colbert possessed complete records of French foreign commerce, nor did they think it worth while to ask for them.

* Mr. Thomas Peregrine Courtenay was vice-president of the Board of Trade from 1828 to 1830 and a member of Parliament from 1810 to 1831.

† Mr. Porter was one of the Founders of the Royal Statistical Society, and the author of *The Progress of the Nation*, a statistical survey of the United Kingdom which was brought up to date and republished in 1912 by Mr. F. W. Hirst, editor of the London *Economist*.

‡ Parliamentary Papers, 1833, Vol. 41, Part I, p. 48, pp. 65 *et seq.*

§ Parliamentary Papers, 1833, Vol. 41, Part II, pp. 20 *et seq.*

‖ Vol. II, p. 118.

According to Monsieur Faure* it was Ponchartrain, Colbert's successor, who first asked in 1693 that detailed tables of importation and exportation be drawn up. However, the Bureau appointed to this task was not created until 1713. Arnould, who was at one time Chief of the Bureau de la Balance du Commerce, and who published in 1795 a lively account of the uses of statistics in government administration,† says that the detailed records of French imports and exports were begun as a result of the controversy in England at the time of the Peace of Utrecht.

From the time that William of Orange became ruler of England, trade between England and France has been rendered practically impossible by the restrictions and tariffs prevailing in both countries. Arnould says:‡

"The Peace of Utrecht in 1713 gave birth to the hope of seeing the French and British markets opened to the merchants of the two nations. In fact, there was drawn up and signed on April 31 of that year, a treaty of commerce and navigation between the two powers. Immediately there arose in England two parties, one for retaining and one for rejecting these articles. The tables of importations and exportations between France and Great Britain were opened and commented upon in public discussions and the general opinion manifested caused the British Parliament to reject the treaty. It was at this period that the French Government, deprived of means of its own of combating the objections of the British commissioners and wishing to determine the advantage to their country of the articles which remained in force in the treaty, by an order in council of April 18, 1713, confirmed by a decree of June 16, 1716, confided to Sieur de Grandval the preparation of tables of importations and exportations from the information assembled in all the French customs houses. This was the origin of the 'Bureau de la Balance du Commerce' in France."

The "public discussions" to which Arnould refers were carried on in the columns of two English magazines, *The Mercator* and *The British Merchant*. Daniel Defoe described in *The Mercator* the beneficial character of the trade with France, and his opponent, in *The British Merchant*, charged "that as they who employed him (Defoe) and furnished him with Materials, had command of all the public papers in the Custom House, he had it in his Power to do a great deal of Mischief, among such as were unskilled in Trade, and, at the same time, very fond of French wine."§ As Arnould points out, "the British Merchant's" policy carried the day, and the almost prohibitive duties upon French goods remained.

The French records of foreign trade begun in 1716 were, like all other French state papers of that time, kept secret. No statement of French

* *History of Statistics*, p. 268.

† *La Balance du commerce et des relations commerciales exterieures de la France* par le Cn. Arnould, Chef du ci-devant Bureau de la Balance du Commerce, l'an 3e de la Republique.

‡ *La Balance du commerce*, p. 121, *et seq.*

§ Cunningham, *opus cit.*, pp. 462–3.

imports and exports was published until the Third Year of the French Republic, when Citizen Arnould, the last chief of the Bureau of the Balance of Trade under the old régime, printed foreign trade figures for France from 1716 to 1788 as an appendix to his book, *The Balance of Trade.** Arnould's figures are judged by French statisticians to be authoritative,† particularly as they are substantiated by statistics published in the August, 1830, number of the *Bulletin de la Société Française de Statistique Universelle.* This Bulletin gives‡ a recapitulation, "prepared from official documents by M. Cesar Moreau," of the value of French foreign trade from 1716 through 1829, with the exception of the years of the Revolution, 1789—97. From 1716 through 1789, the figures represent simply the value of the merchandise and specie imported and exported, and the trade of no one year is given separately, the years 1716 through 1720 being grouped together, and so on. A preliminary note states that "the figures for the first sixty years are taken from a manuscript belonging to M. Moreau, which was drawn up for the information of the King, Louis XVI, by M. Trudaine de Montigny, Chancellor of State, a magistrate distinguished for his learning, ability, and accuracy. The rest of the material is drawn from the best sources and for the last years is based upon official documents." For the years 1787, 1788, and 1789, and for each year from 1797 through 1829, the table presents the value of the specie and merchandise imported into and exported from France, giving separate figures for the trade in raw materials and manufactured articles.

More detailed figures on early French trade are published in the *Commerce Exterieur*, the seventh volume of the *Statistique de la France* published by Le Ministre des Travaux Publics, de L'Agriculture et du Commerce in 1838. This publication gives (a) the value of the imports and exports of France by countries for 1787, 1788, 1789, 1792, and for each year from 1797 to 1810; (b) the value of the "general" commerce and the "special" commerce from 1815 to 1837, with separate figures for (1) the producers' and consumers' goods imported, separating the latter into natural and manufactured articles, and (2) for the raw materials and manufactured articles exported. In most cases where the figures given by the Department of Commerce cover the same years as those of M. Arnould and M. Moreau, they differ by several thousand francs. M. Faure is of the opinion that the unofficial

* I do not know how detailed Arnould's figures are. The only copy of *La Balance du Commerce* which I have been able to find is the one belonging to the Columbia University Library, and it lacks the third volume, the "Atlas" in which the tables are printed.

† *The History of Statistics*, p. 270.

‡ Pp. 12–13.

figures are to be preferred. In 1823 the Direction Generale des Douanes was given the task of publishing trade statistics for France. The report of the annual trade for 1820 was the first volume to appear in this series which has been unbroken ever since.

The progress of foreign trade statistics was not rapid in the eighteenth century, largely because of unsettled political conditions in Europe. Aside from England and France, only three countries had begun the regular compilation of their foreign trade statistics before the beginning of the nineteenth century—Portugal, Spain, and the United States of America.

In 1822 M. Adrien Balbi addressed to the King of France an *Essai Statistique sur la Royaume de Portugal et D'Algarin* in which he tells of his researches in Portuguese archives. "What was our astonishment to find," says he, "in a country hardly as advanced as Turkey, a general statement of commerce prepared annually from 1775 to the present by Chevalier Mauricio Teixeira Moraes, after a plan and with an exactitude that would be hard to find in the most civilized countries of Europe!"* In the course of his account of Portuguese industrial life, Balbi quotes tables of Portuguese foreign trade in 1796, 1806, and 1819, classified by countries and groups of commodities and expressed in value units, which are very satisfactory summary tables.†

When we consider how much of Spain's early wealth was the result of trade with her American colonies, it seems strange that she did not begin earlier to tabulate statistics on her overseas trade. Her method of levying customs duties varied greatly, but from 1624 to 1778, no declaration of exports from shippers was required and duties were levied on either the weight or the cubic measurement of the goods to be shipped, or the number of pieces in the cargo! Mr. C. H. Haring, writing in the Harvard Economic Series of *Trade and Navigation Between Spain and the Indies*, says of this rather remarkable practice:‡

"There is also a suspicion that there may have been another reason (than the desire to avoid unnecessary delay in shipments) for the opposition to a more exact appraisal of goods passing through the customs, for the Crown's willingness to trust to the good faith of the shipper. It was inconvenient to the Seville monopolists that exact knowledge of the cargoes sent to the great fairs at Porto Bello and Vera Cruz should become public. For, as the colonies were chronically undersupplied, the general level of prices at these fairs was governed by the comparatively simple procedure of balancing the whole value of the goods brought out with the whole amount of raw materials and bullion offered by the Creole merchants for exchange. And the Castilian exporters, by manipulating each year the nature and the quantity of the merchandise to be shipped to the Indies, raised prices at will and reaped enormous profits,

* Vol. I, p. viii.
† Vol. I, p. 442.
‡ P. 91.

said to have amounted to 300 or 400 per cent. It was a matter of some concern, therefore, that the character of the shipments should not be made known till the fleets reached their destination. And as the King was usually under obligation to the wealthy monopolists for gifts and loans to the exchequer, he was more apt to connive at practices of this sort."

Mr. Haring says* that this method of assessing customs duties remained in force until 1778. I judge that very shortly after that date the Spanish government began the regular compilation of the commodities entering into its foreign trade, for Alexandre Moreau de Jonnes, in his *Statistique de L'Espagne*, published in 1834, gives foreign trade figures for Spain from 1784 on, which came, he says, from unpublished official documents.†

Since the causes of the American Revolution, economic as well as political, were intimately bound up with foreign trade, it is not surprising that the second act of the first Congress of the United States authorized the Secretary of the Treasury to compile complete statistics of imports and exports. This is the more easily understood since the new government purposed to secure a good part of its revenue from taxation of the commodities imported into the country.

The first printed results of this act of 1789 were presented to the House of Representatives on February 15, 1791, by Alexander Hamilton as "a general return of the exports of the United States, abstracted from custom house returns, commencing on the various days in August whereon they were respectively opened, and ending on the 30th of September last." Hamilton was inclined to be apologetic about the period of time covered in the return. In his letter of transmittal to the speaker of the House, he wrote:‡

"From inadvertence in some of these (custom) offices, the space of time prior to the 1st of October, 1789, was blended with the quarter following, which prevented a uniform commencement of this abstract on that day; and yet there is a deficiency for the last quarter of 1790 which confines the abstract to the 30th of September last."

In this first account of exports from the United States there is stated the total quantity and the total value of each commodity exported by countries, and the total value of the export trade to each country. The commodities enumerated are arranged partly in alphabetical order, the first five being

Potash
Pearlash
Apples
Boats
Bombshells

* P. 92.

† "Nous en empruntos les chiffres a des documens officiel inedits." P. 159.

‡ American State Papers, Vol. VII, p. 23.

and partly in groups, as

Fishery	Grain	Ginseng
Fish, dried pickled	Buckwheat	Gunpowder
Oil, whale spermaceti	Corn	Gin
Candles	Oats	Grindstones
Whalebone	Rice	Hair Powder (*)
	Wheat	

In this fashion there are presented the exports of 136 commodities to the following places:

Great Britain	Dutch West Indies
France	Spanish West Indies and Florida
Spain	Swedish West Indies
Portugal	Portuguese Islands
Holland	African Islands and the Coast of Africa
Flanders	East Indies (with an undivided total of ten thousand three hundred sixty-two dollars' worth to "the Northwest Coast of America")
Germany	
Denmark	
Sweden	
Mediterranean	
Nova Scotia	
British West Indies	
French West Indies	

The statement of imports into the United States for the year ending September 30, 1790, communicated to the Senate on November 18, 1791, is clearly the product of the tax-gathering organization of the government. It presents imports into the United States from thirty separate geographical divisions, distinguishing commodities from "England, Man and Berwick" from those from "Guernsey, Jersey, Sark and Alderney," but reporting those from "Africa generally" together. No details are given as to goods subject to duty ad valorem, other than the value of such goods from each country; and to the "enumerated goods paying a specific duty," which are stated in quantity units, no value is assigned. It is therefore impossible to arrive at the total value or the total quantity of imports into the United States for that year. Although a great many tables on foreign trade were submitted to Congress in the interim,† the first attempt to present the annual

* American State Papers, Vol. VII, pp. 24–34.

† Under the title "The Statistical Work of the Federal Government," Julius H. Parmelee published in the *Yale Review* for November, 1910, the following unofficial sources of information on this period:

1. *A View of the United States of America*, published by Tench Coxe in 1794, giving official documents for 1790, 1791, and 1792.

2. *A Statistical Manual for the United States of America*, by Samuel Blodget, Jr., Washington, 1806.

3. *A Statistical View of the Commerce of the United States of America*, by Timothy Pitkin, Hartford, 1816. Many of the tables in this collection were obtained directly from the Treasury books, and had never been published.

4. *Statistical Annals* by Dr. Adam Seybert, covering the period 1789–1818, and giving statistics of population, commerce, public land, etc. Five hundred copies of this work were bought by Congress.

import and export statistics of the United States to the public in detailed and systematic fashion was made in February, 1821, when an act which provided for "obtaining accurate statements of the foreign commerce of the United States" was passed.

The first volume of the new series was entitled *Commerce and Navigation of the United States for the Year ending September 30, 1821.* It too was very evidently prepared by men whose interests were fiscal rather than commercial. The commodities entering into the import trade are classified by countries, and grouped as follows:

1. Value of merchandise free of duty
2. Value of merchandise paying duties ad valorem
3. Quantity and value of merchandise paying specific rates of duty

"The Goods, Wares, and Merchandise of the Growth, Produce and Manufactures of the United States Exported" are reported in quantity and value units, and classified by countries.

There had been no break in the series of annual foreign trade statistics for the United States since 1821, and there has been some improvement in the compilation and publication of the facts involved. In the years 1843–44, the fiscal year was changed from the year ending September 30th to that ending June 30th, and beginning with the volume for 1918, United States trade statistics have been compiled for the calendar rather than the fiscal year. The method of classifying the commodities exchanged has been radically reformed.

By the middle of the nineteenth century the publication of national import and export statistics had become the rule rather than the exception. In the Netherlands, a royal decree of 1825 called for the regular collection of data concerning imports, exports, and goods in transit, and commercial statistics were included in the Census of 1829.* Figures for Belgian trade date from the year of its winning independence; the first Belgian statistics published record trade facts for the years 1831–34.† Norway began the compilation of import and export statistics at the beginning of the nineteenth century, although no complete tables were published before 1835.‡ The Revenue Department of the Russian Empire also began to record trade figures not long after 1800, and early published a volume called *Foreign Commerce of Russia in its Principal Aspects.*§ In Denmark annual publication of foreign trade figures began in 1854,‖ and in Sweden in 1858.¶

* *History of Statistics*, pp. 432–33.
† *Ibid.*, p. 143.
‡ *Ibid.*, p. 448.
§ *History of Statistics*, p. 509.
‖ *Ibid.*, p. 203.
¶ *Ibid.*, p. 556.

Until after 1871 the organization of trade statistics in Germany was difficult because of the lack of national unity. Some of the states of the Confederation of the Rhine published trade figures as early as 1812,* and the formation of the German Tariff Union in 1833, by its very nature necessitated trade statistics. It was natural, therefore, that very shortly after the formation of the German Empire (on March 9, 1872) a resolution was adopted founding an Imperial Statistical Office to prepare, among other things, statistics of foreign trade.†

Austria-Hungary began the annual compilation of *Tables of Statistics* (including figures on foreign trade) with the year 1828, but kept them secret until after the revolution of 1848–49. The 1850 volume of the *Tables*, which was made public property, includes trade statistics for the years 1845 and 1846.‡

In 1862 the Northern States of Italy were publishing trade figures for 1859, and the scope of these figures was gradually enlarged with the growth of the Kingdom of Italy.

In 1859 China began the publication of statistics on the Ports opened by Treaty, and in 1872 the first of the Japanese Political Tables were printed. Since England, France, and Germany (and in the case of the Philippines and Hawaii, the United States) have interested themselves in the compilation of trade statistics for most parts of Asia, Africa, and Oceania, it is safe to say that by the year 1900 the trade of almost every country in the world was being recorded in some fashion each year.

* See *Statistik der Rheinbundstaaten*, Vol. I, pp. 323–25, and Vol. II, p. 262, pp. 240–41.
† *History of Statistics*, p. 337.
‡ *Tafeln zur Statistik*, 1850, Vol. II, Sec. 5.

A NEW MEASURE OF DISPERSION*

By Truman L. Kelley, *Stanford University*

The measures of dispersion in common use are: (1) the standard deviation; (2) average deviation; (3) quartile deviation; and (4) the range. It is generally admitted that the merit of the range as a measure of dispersion lies in its simplicity of determination and interpretation. To calculate its probable error has not been attempted; but were this done it would have little significance, for the range itself is not a constant function of a distribution, since, in general, it increases with an increase in population. The range is therefore unsatisfactory as a measure of dispersion, because it undoubtedly has a large probable error, and also because its significance varies as populations of different sizes are being considered.

The probable errors of the other three measures are given in the second column of the accompanying table. The values in columns 2 and 3 must be multiplied by the standard deviation of the distribution, and the values in columns 2 and 4 must be divided by the square root of the number of cases ($\sqrt{n}$) before they will serve in any particular problem. For purposes of comparison of different measures, these factors are here omitted.

TABLE

1 Measure of dispersion (D)	2 Probable error of measure of dispersion ($P.E._D$)†	3 Measure of dispersion in terms of the standard deviation‡ Range covered by measure in case of a normal distribution	4 The probable error of the measure of dispersion divided by the measure of dispersion§ $\left(\frac{P.E._D}{D}\right)$	5 Population necessary to secure a reliability equal to that of the standard deviation when determined from 100 cases
Standard deviation	0.4769*	1.0000	0.4769‡	100
Average deviation	0.4066	0.7979†	0.5096	114
Quartile deviation	0.5306	0.6745	0.7867	272
Range between percentiles:				
49 and 51	.....	.....	4.721	9798
45 and 55	.....	.....	2.036	1822
40 and 60	.....	.....	1.378	835
35 and 65	.....	.....	1.082	515
30 and 70	.....	.....	0.906	360
25 and 75	.....	1.3490	0.787	272
20 and 80	.....	.....	0.702	216
10 and 90	.....	2.5631	0.6001	158
7 and 93	.....	2.9640	0.5905	153
5 and 95	.....	3.2897	0.5964	156
2 and 98	.....	.....	0.665	196
1 and 99	.....	.....	0.756	251

* Credit is due to William Betz and W. J. Osborn for the calculation of several of the probable errors reported in this article.

† The values in column 2 should be mutiplied by σ and divided by $\sqrt{n}$ to give the full definition.

‡ The values in column 3 should be multiplied by σ.

§ The values in column 4 should be divided by $\sqrt{n}$.

The formula used for the calculation of the probable error of (1) the standard deviation is $0.6745\frac{\sigma}{\sqrt{2n}}$. The following formulas for the probable errors of (2) the average deviation and (3) the inter-percentile range have been derived by the writer from elementary principles:

(2) Probable Error of the average deviation equals $0.5096\frac{A.D.}{\sqrt{n}}$.

(3) Probable Error of the inter-percentile range equals

$$0.6745\sqrt{\frac{np_1q_1}{y_1^2}+\frac{np_2q_2}{y_2^2}-\frac{2np_1q_2}{y_1y_2}}$$

in which p_1 and p_2 are the proportions lying below the percentiles studied; *e. g.*, if the probable error of the inter-quartile range is being determined, $p_1=0.25$ and $p_2=0.75$ (p_1 being always the smaller of these two proportions). The q's are defined by the usual relationship $p_1+q_1=1$. Y_1 is the ordinate of the distribution at the p_1 percentile, and may be directly read from the distribution (preferably after slight smoothing), but if the normal distribution is assumed, it can be determined from Sheppard's tables of the ordinates of the normal probability integral.

In case it is desired to obtain the probable errors of distances between percentiles leaving equal amounts of area at each end of the distribution, the formula becomes for the normal distribution, $P.E._D=\frac{0.6745\sigma}{z\sqrt{n}}\sqrt{2p_1q_1-2p_1^2}$ in which D is the distance between the percentiles $100p_1$ and $100\ (1-p_1)$, σ is the actual standard deviation of the distribution, and z is the entry for the ordinate in Sheppard's tables corresponding to the proportion p_1. The formula for the probable error of this inter-percentile range is not in convenient shape for actual calculation, for it contains σ, which is usually unknown. As, however, the relationship between D and σ in the case of a normal distribution can be immediately obtained by referring to Sheppard's tables for any proportion, p, the formula can be readily transformed into one for practical application. In case $p_1=.10$, so that D is the distance between the 10th and 90th percentiles, this formula becomes $P.E._D=\frac{0.6001}{\sqrt{n}}D$. Values for other inter-percentile ranges are given in column 4 of the table.

Empirical calculation in the case of a normal distribution of the probable errors of inter-percentile ranges in terms of their own magnitudes

(such quantities as 0.6001 above) shows that the most reliable D for a given percentage difference, *i. e.*, for a given $p_2 - p_1$ magnitude, is to be obtained when equal proportions are cut off at each end of the distribution, or, in other words, when $p_2 = 1 - p_1$. Accepting this empirical finding as evidence of universal truth, the writer examined analytically ranges equally curtailed at the extremes. $\frac{P.E._D}{D}$ is the function which is desired to make a minimum. Differentiating this and setting the derivative equal to 0 and solving, results in the value $p_1 = 0.06917$ and $\frac{P.E._D}{D} = \frac{0.59053}{\sqrt{n}}$. Accordingly, if a distribution is normal, the most reliable inter-percentile range is found to be the range between 6.917 and 93.083. An examination of column 4 shows that the probable error of this range is not so much greater than that of the standard deviation as to make it an unserviceable measure, especially for rapid investigation or when percentiles must be calculated in any case.

In place of this 7 to 93 percentile range, practically nothing of reliability is lost if a slightly different range is used. Because of the very fortunate constant 0.6001, and because of the general necessity of knowing both the 10th and 90th percentiles if any percentiles at all are needed, it is herewith pointed out that the 10 − 90 percentile range could to advantage be used in place of the semi-inter-quartile range or quartile deviation as a measure of dispersion. The relation in a normal distribution between this range and the quartile deviation is expressed by the equation $Q = 0.26315\ D$; between this range and the standard deviation $\sigma = 0.39015\ D$.

The properties of the 10 − 90 percentile range here reported are based upon a normal distribution, but an examination of the general formula for the probable error of an inter-percentile range suggests that this 10 − 90 percentile range is superior to the quartile deviation for practically all distributions. Comparatively it is more excellent for leptokurtic or peaked distributions, and slightly less excellent, though still much more reliable than the quartile deviation, for platykurtic or flat topped distributions.

The 10 − 90 percentile range may readily be used as a basis for determining the existence of skewness and the nature of the kurtosis of a distribution. If the distribution is not skew, the average of the 10th and 90th percentiles will be equal to the median. The difference between the average of these two percentiles and the median is thus a measure of skewness. The probable error of this difference is equal to

$0.40412\frac{D}{\sqrt{n}}$. If the difference is small with reference to its probable error, skewness is not established.

Further, for a mesokurtic distribution, $Q=0.26315D$. Therefore, if $\frac{Q}{D}$ is <0.26315, the distribution is leptokurtic, and if $\frac{Q}{D}$ is >0.26315, the distribution is platykurtic. The probable error of $\frac{Q}{D}=\frac{0.18736}{\sqrt{n}}$.

The two formulas just given are based upon a normal distribution.

Appendix.

Derivation of formulas giving probable errors.

Let σ equal the standard deviation of the measure designated by the subscript.

σ without a subscript will stand for the standard deviation of the original distribution dealt with.

Let p equal the proportion of cases below a certain value P of the variate; accordingly the value of the $100p^{th}$ percentile is P.

Let q be defined by the usual relation $p+q=1$.

Let $Q=\frac{P_{75}-P_{25}}{2}$.

Let $D_{100p'-100p}=P_{100p}-P_{100p'}$.

Let D without subscript $=D_{10-90}=P_{90}-P_{10}$.

The following formulas are well known:

$$\sigma_p=\sqrt{\frac{pq}{n}}$$

$\sigma_P=\frac{1}{y_p}\sqrt{npq}$, in which y_p is the ordinate of the distribution at P, the $100p^{th}$ percentile.

$$\sigma_\sigma=\frac{\sigma}{\sqrt{2n}}$$

$r_{PP'}=\sqrt{\frac{pq'}{p'q}}$, in which $p<p'$.

Appendix A.

The calculation of the probable error of the average deviation.

The average deviation, or more exactly, the mean deviation, is the first moment of a distribution obtained by treating all the deviations from the mean of an original distribution as though positive. The

mean deviation is thus the mean of a distribution of form like the half of a normal distribution and its standard deviation is to be obtained in exactly the same manner as the standard deviation of any other mean. Accordingly, $\sigma_{A.D.}=\sqrt{\frac{\bar{\mu}_2-\bar{\mu}_1^2}{N}}$, in which $\bar{\mu}_1$ and $\bar{\mu}_2$ are the first and second moments around an arbitrary origin. If moments around the stump of the half distribution are taken, they are moments around an arbitrary origin so far as the half distribution is concerned, though they are also moments around the mean of the total normal distribution. Accordingly $\bar{\mu}_2=\sigma^2$ and $\bar{\mu}_1=\frac{y_0\sigma}{.5N}$, in which y_0 is the ordinate of the normal curve at the mean. $\frac{y_0}{N}$ as found in Sheppard's tables $=0.3989$. Substituting and simplifying yields:

$$\sigma_{A.D.}=\frac{0.6028\sigma}{\sqrt{N}}=\frac{0.7555(A.D.)}{\sqrt{N}}$$

$$P.E._{A.D.}=\frac{0.5096(A.D.)}{\sqrt{N}}.$$

Appendix B.

The calculation of the probable error of an inter-percentile range.

$$\sigma D_{100p-100p'}=\sqrt{\sigma_P{}^2+\sigma_{P'}{}^2-2r_{PP'}\sigma_P\sigma_{P'}}=\sqrt{\frac{npq}{(y)^2}+\frac{np'q'}{(y')^2}-\frac{2npq'}{yy'}}.$$

If a normal distribution is assumed, each y is known when p is determined, and if $n=1$ and $\sigma=1$, y then becomes the ordinate z recorded in Sheppard's tables.

The reliability of a measure of dispersion is significant in terms of the magnitude of the measure, so that if the particular values of p and p' can be determined for which $\frac{\sigma_{D_{100p-100p'}}}{D_{100p-100p'}}$ is a minimum, the most reliable inter-percentile range will be found. Calculation of such ratios for a normal distribution shows that for a D of a given magnitude, or for a given $(100p-100p')$ magnitude, the ratio is always a minimum when $p=1-p'$. The writer has not proved this analytically, but empirical results warrant the generalization. It thus remains to determine the value of p for which the function, $f=\frac{\sigma_{D_{100p-100p'}}}{D_{100p-100p'}}$ is a minimum.

Appendix C.

The determination of the inter-percentile range having the minimal error.

It is desired to determine the value of D for which the function just given is a minimum. If x equals a deviation from the mean, then, assuming a normal distribution, $D=2x$, and the solution of the equation $\frac{df}{dx}=0$ will give the value of x and therefore of D and p for the inter-percentile range having the minimal error.

$$f=\frac{\sqrt{\frac{npq}{y^2}+\frac{np'q'}{(y')^2}-\frac{2npq'}{yy'}}}{2x}$$

$$y=\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}};\quad p=\int_{-\infty}^{x}ydx;\quad n=1$$

y is the value tabled as z for different x's and p's in Sheppard's tables. If equal amounts are cut off at each end of the distribution then: $p=1-p'$; $p=q'$; $q=p'$; and $y=y'$; so that,

$$f=\frac{\sqrt{2p-4p^2}}{2xy}$$

$$\frac{df}{f}=\frac{d\sqrt{2p-4p^2}}{\sqrt{2p-4p^2}}-\frac{d(2x)}{2x}-\frac{dy}{y}.$$

Noting that $p<.5$, that $\frac{dp}{dx}=$y, and that $\frac{dy}{dx}=-xy$, the differential equation becomes:

$$\frac{df}{dx}=f\left[\frac{(1-4p)y}{2p-4p^2}-\frac{1}{x}+x\right].$$

Setting this derivative equal to zero and solving by the aid of Sheppard's tables yields $p=0.06917$, so that the minimal error inter-percentile range is that between the 6.917 and the 93.083 percentiles.

Appendix D.

The calculation of the probable error of the difference between the P_{50} and the average of the P_{10} and P_{90} percentiles.

Let $f=P_{50}-\frac{1}{2}(P_{90}+P_{10})$

$\Delta f=\Delta P_{50}-\frac{1}{2}\Delta P_{90}-\frac{1}{2}\Delta P_{10}$.

Squaring, summing, and as a first approximation setting $\sigma_f^2=\frac{\Sigma(\Delta f)^2}{N}$,

$\sigma_{P_{50}}^2 = \frac{\Sigma(\Delta P_{50})^2}{N}$, etc., gives,

$$\sigma_f^2 = \sigma_{P_{50}}^2 + \tfrac{1}{4}\sigma_{P_{90}}^2 + \tfrac{1}{4}\sigma_{P_{10}}^2 + \tfrac{1}{2}r_{P_{90}P_{10}}\sigma_{P_{90}}\sigma_{P_{10}} - r_{P_{50}P_{90}}\sigma_{P_{50}}\sigma_{P_{90}} - r_{P_{50}P_{10}}\sigma_{P_{50}}\sigma_{P_{10}}.$$

Simplifying, $\sigma_f = 1.53567\ \frac{\sigma}{\sqrt{n}} = 0.599143\ \frac{D}{\sqrt{n}}$

$$P.E._f = 0.40412\ \frac{D}{\sqrt{n}}.$$

Appendix E.

The calculation of the probable error of the quotient $\frac{Q}{D}$.

Let $f = \frac{Q}{D}$

$$\frac{df}{f} = \frac{dQ}{Q} - \frac{dD}{D}$$

$\frac{\sigma_f^2}{f^2} = \frac{\sigma_Q^2}{Q^2} + \frac{\sigma_D^2}{D^2} - 2r_{QD}\frac{\sigma_Q\sigma_D}{QD}$, in which r_{QD} may be shown to $= 0.5$. Simplifying,

$\sigma_f = 0.27779\frac{1}{\sqrt{n}}$, or

$$P.E._{\frac{Q}{D}} = 0.18736\frac{1}{\sqrt{n}}.$$

OCCUPATIONS OF IMMIGRANTS BEFORE AND AFTER COMING TO THE UNITED STATES

By Louis Bloch*

INTRODUCTION

In this article are set forth the results of an effort to learn the extent to which the immigrant, after coming to this country, utilizes his previous experience as an industrial or agricultural worker. Does a carpenter from the old country, for instance, work in this country as a carpenter, or does he find employment in some other occupation in which he cannot utilize his trade experience? Similarly, to what extent do agricultural laborers and farmers from the old country find agricultural employment in this country? As will be seen from the following pages, the problem does not lend itself to an easy solution.

The data used in the investigation were drawn from the following sources: (1) the reports of the United States Immigration Commission; (2) the United States censuses of 1880, 1890, 1900, and 1910; and (3) the reports of the Commissioner General of Immigration.

One of the difficulties encountered in the use of the census data was a perplexing lack of uniformity of occupational designations. To make matters somewhat more complicated, the occupations of arriving immigrants, as given in the reports of the Commissioner General of Immigration, are not classified in such a way as to make them readily comparable with the occupation groups of any of the censuses referred to. Owing to this lack of uniformity in classification of occupations, it was found to be impossible to include more than a few occupations.

In studying the data presented below, it should be remembered that the number of arriving immigrants reported does not represent the *net* increase of immigrants in the respective occupations. To secure the net increase it is necessary to deduct from the total arrivals the total number of immigrants in the respective occupations who departed during the decades studied. Owing to the fact that no figures for departing emigrants appear in the reports of the Commissioner General of Immigration until 1908, it is impossible to get at the exact increase of the occupational groups studied. Beginning with 1908, the number of departing immigrants, by occupation, is

* In the preparation of this article the writer received valuable suggestions from Dr. C. C. Williamson, Statistician for the Carnegie Americanization Study. This article was originally prepared for the members of the staff of the Carnegie Americanization Study and appeared as Bulletin No. 32 (mimeographed) of its Division of Information.

given annually in the reports of the Commissioner General of Immigration.

It was found desirable to divide the study into two sections, one dealing with agricultural laborers and farmers, the other with skilled industrial workers.

SECTION I

FOREIGN-BORN AGRICULTURAL LABORERS AND FARMERS IN THE UNITED STATES

It is generally conceded to be a fact that a considerable majority of immigrant agricultural laborers and farmers do not find employment in agricultural pursuits in the United States. A statement to the effect that not one in ten immigrant agricultural laborers settles on a farm in this country is found in the *Immigration Problem* by Jenks and Lauck (edition of 1917, p. 100). In his report on immigration to the United States Industrial Commission of 1900, Professor Commons says it is certain that a large proportion of immigrant *laborers* and *farm laborers* destined for Pennsylvania in 1900 found employment in the mines of that state.*

The following table (Table I) shows that in the year 1900 there were 259,294 foreign-born whites of both sexes engaged as farm laborers in this country. In 1910 the number of foreign-born white farm laborers was 336,753, an increase of 77,459 or 29.9 per cent. According to the reports of the Commissioner General of Immigration, 1,602,743 immigrant agricultural laborers, male and female, arrived in the United States between 1901 and 1910, both dates inclusive. From 1908 to 1914, however, 1.9 per cent as many farm laborers went back as came in, according to the report of the Commissioner General of Immigration. On this basis the net addition to the farm laborers may be estimated at 1,572,291. If all of these had actually engaged in agricultural pursuits in this country, the increase would have been 606 instead of 29.9 per cent. The per cent of farm laborers departing again must have been much larger than reported. The occupation given on departure is apparently that followed in this country, so that many whose training and experience in the old country were in agriculture return as skilled or unskilled workers in other occupations. In the years 1908–1914 the total number of departing immigrants amounted to 36.2 per cent of all arrivals. For skilled occupations, as indicated in Table V, the proportion of those going back is only about one-third as great (13.3 per cent for sixteen skilled occupations) as for all occupations. It is evident, therefore, (1) that the per cent

* Vol. 15, p. 389.

of farm laborers going back to the old country is larger than the per cent of those returning in all occupations, and (2) that returning immigrants report the occupation they followed in this country. Instead of 1.9 per cent of those who came in as farm laborers returning to the old country, an estimate of 50 per cent would probably be conservative. Assuming, therefore, that as many as half of the agricultural laborers returned, the increase in foreign-born farm laborers would have been over 300 per cent instead of 29.9 if they had all followed their former occupations. The actual increase was 77,459; the net addition by immigration is estimated at 801,371. Consequently, it would seem that not more than 10 per cent of immigrant farm laborers go into agricultural pursuits in this country.

TABLE I

NUMBER OF FOREIGN-BORN WHITES ENGAGED AS FARM LABORERS IN THE UNITED STATES, BY SEX: 1900 AND 1910

(U. S. Census of 1910, vol. IV, p. 303; U. S. Census 1900, Special Report on Occupations 1904, p. 11)

Sex	1910	1900	Increase 1900 to 1910
Males	308,360	253,895	21.5
Females	28,393	5,399	425.9
Both sexes	336,753	259,294	29.9

The Immigration Commission made a detailed study of 17,141 households, the heads of which were miners or wage-earners in manufacturing establishments. Table II shows that of the persons of these households for whom complete data were secured, 62 per cent of the males and 24.3 per cent of the females were employed as farm laborers or as farmers before coming to the United States. Table III shows

TABLE II

OCCUPATIONS BEFORE COMING TO THE UNITED STATES OF FOREIGN-BORN MALES AND FEMALES WHO WERE 16 YEARS OF AGE AND OVER AT TIME OF COMING

(From Reports of Immigration Commission, vol. 19, pp. 89, 93, Study of Households)

Number reporting complete data	Per cent without occupation	Per cent working for wages				Per cent working without wages		Per cent working for profit	
		Farm laborers	General laborers	Domestic service	All other occupations	Farm laborers	All other occupations	Farmers	All other occupations
Males									
21,696	3.2	20.3	2.9	....	27.8	28.6	.9	13.1	3.0
Females									
13,315	59.7	7.3	....	8.4	6.8	16.4	.5	.6	.4

that of the members of the same households for whom complete data were secured, 0.1 per cent of the males and less than 0.05 per cent of the females were employed in agricultural pursuits in this country. It should be remembered, however, that the households studied by the Immigration Commission were those whose heads were engaged as miners or wage-earners in manufacturing establishments; consequently, it should not be expected that any considerable proportion of the members of these households would be employed in agricultural pursuits.

TABLE III

PER CENT OF FOREIGN-BORN MALES AND FEMALES 16 YEARS OF AGE AND OVER ENGAGED IN SPECIFIED INDUSTRIES

(From Reports of Immigration Commission, vol. 19, pp. 101–102, Study of Households)

Number reporting complete data	Agricultural pursuits	Domestic and personal services	Manufacturing and mechanical pursuits	Mining	General labor	Professional services	Trade	Fishing	At home	At school	Transportation	Total
Males 25,727	.1	.8	67.9	25.0	1.3	.1	1.1	(*)	2.5	.2	1.0	100.0
Females 17,117	(*)	2.1	16.5	(*)	(*)	(*)	.5	.0	80.3	.4	(*)	99.8

* Less than 0.05 per cent.

The Immigration Commission also secured detailed information from groups of employees in mines and manufacturing establishments. Table IV shows that of the 181,330 male employees reporting complete data the larger proportion, about 54 per cent, were employed in the old country in farming or as farm laborers, while of the 12,968 female employees reporting complete data, 44 per cent were employed in the old country in farming or as farm laborers.

TABLE IV

PER CENT OF FOREIGN-BORN EMPLOYEES IN SPECIFIED OCCUPATIONS BEFORE COMING TO THE UNITED STATES

(From Reports of Immigration Commission, vol. 19, pp. 95–98, Study of Employees)

Number reporting complete data	In manufacturing	Farming and farm labor	General labor	Domestic service	Sewing, embroidery and lace making	Teaching	Trade	Other occupations	Total
Males 181,330	15.3	53.9	10.3	....	....	....	2.5	18.1	100.1
Females 12,968	32.5	44.2	....	6.2	13.4	.7	1.8	1.4	100.2

SECTION II

HOW SKILLED IMMIGRANT WORKERS ARE EMPLOYED IN THE UNITED STATES

The data presented in the first part of this article show that a considerable majority of immigrant agricultural laborers and farmers do not follow their former occupations after coming to the United States. It was shown that of the members of the households studied by the Immigration Commission 62 per cent of the males and about 24 per cent of the females had been engaged as agricultural laborers and farmers in the old country; while in this country they were mainly employed in manufacturing and mining. It is apparent that any skill or experience in agricultural pursuits which these immigrants may have possessed is largely unutilized in this country. An analysis of the available data relating to skilled immigrant workers shows that a similar situation exists with regard to this class of foreign-born labor.

Table IV, based upon a study of employees engaged in this country in manufacturing and mining, shows that only 15.3 per cent of the males and 32.5 per cent of the females had had any experience in manufacturing before coming to the United States. It is evident, therefore, that about 84 per cent of the males and about 67 per cent of the females in this group were not utilizing such skill and experience as they may have acquired in the occupations they had been engaged in before coming to the United States.

The difficulties met with in attempting to compare the census data for specific occupations for the years 1900 and 1910 have been referred to above. After making a careful examination of all the occupational groups, sixteen have been selected for which the figures seem to be reasonably comparable for the two censuses. For these sixteen occupations, therefore, an effort has been made to learn what light the available statistics throw on the utilization of the immigrant's industrial skill and experience.

The method adopted is to compare the increase or decrease in the number of foreign-born reported for each occupation in 1910 with the number of immigrants belonging to the same occupation who arrived during the decade, as reported by the Bureau of Immigration. The result at best can be regarded as only approximately accurate. It is evident, however, that it is those who remain in the United States and not the total number of those who arrive that must be regarded as the new additions in each occupation.

As pointed out above, statistics of departures are not available prior to 1908, so that the number of foreign-born in each occupation returning during the decade can only be estimated. Comparing the figures

for arrivals and departures in each of the selected occupations during the period 1908–14, inclusive, it is found that 13.3 per cent of all who came went back to the old country, but that the per cent of departures varies greatly for the different occupations. Accordingly, in Table V the net addition of foreign-born in each occupation is found by deducting from total arrivals in the decade the per cent of departures in the years 1908–14. This process involves a slight error, varying in importance with the different occupations, since no account is taken of the fact that a certain number of those departing under one occupation classification came in under another. No estimate of the number of such cases can be given. Table V, therefore, assumes that all immigrants reported on departure the same occupation they gave on arrival. The result is that our estimate of the per cent of immigrants who abandon their former occupations (Table V, column 9) after reaching the United States is an under-estimate, for the reason that it includes only those who remained in the country.*

Table V summarizes the available data in regard to (1) the actual increase in the number of foreign-born engaged in each of the sixteen selected occupations, according to the census figures, and (2) the net additions to each of the sixteen occupations which could have resulted from immigrants arriving during the decade 1900 to 1910.

Special attention may be called to the facts revealed in Table V for each of the sixteen selected occupations. The table shows that in 1910 there were 5,964 more foreign-born white bakers than in 1900, an increase of 13.8 per cent. If all the bakers who arrived during the decade 1901–10, less the 15.2 per cent estimated to have returned in the same period, had followed their former occupation, the increase from 1900 to 1910 would have been 69.8 per cent. These figures seem to show that about 80 per cent of the bakers who arrived and remained in the United States between 1901 and 1910 went into other occupations.

In discussing the difficulties of classifying occupations, the census of 1910 (Vol. IV, p. 21) calls attention to the fact that in common with several other occupations, the number of bakers reported is excessive. The reason given is that " . . . in all those cases in which hand trades, through the introduction of machinery, the multiplication of processes, and the division of labor, have rather recently developed into factory industries, there was a marked tendency for the operatives to give the old trade names as their occupations, rather than the names

* This under-estimate is counterbalanced by the fact that no account is taken of the mortality rates in the respective occupations. Assuming a death-rate of twenty per cent in a decade for each occupation, the percentages of immigrants who abandoned their former occupations (Table V, col. 9) would be reduced by about twenty per cent. This, however, would not vitiate the general conclusions arrived at.

TABLE V

FOREIGN-BORN WHITES, BOTH SEXES, ENGAGED IN SPECIFIED OCCUPATIONS IN 1900 AND 1910, INCREASE 1900 TO 1910, AND THE NUMBER OF WHITE IMMIGRANTS ARRIVING AND REMAINING IN THE UNITED STATES DURING 1901–10; AND THE PER CENT ESTIMATED TO HAVE ABANDONED THEIR FORMER OCCUPATIONS

Occupations	Number engaged		Increase (or decrease) from 1900 to 1910		Number arriving 1901–10	Per cent departing 1908–14	Number remaining from those who arrived 1901–10	Number remaining 1901–1910, expressed as per cent of total enumerated in 1900	Per cent of immigrants who abandoned their former occupations, 1901–10
	1900	1910	Number	Per cent					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Sixteen occupations	547,072	725,790	100,518	....	489,503	13.3	424,399	73.9	76.3
Bakers	43,368	49,332	5,964	13.8	35,707	15.2	30,280	69.8	80.3
Barbers and hairdressers	34,595	53,431	18,836	54.4	24,056	20.9	19,028	55.0	1.0
Blacksmiths	62,950	66,956	4,006	6.4	44,637	9.8	40,263	64.0	90.1
Bookbinders	5,911	5,356	−555	−9.4	4,359	5.4	4,124	69.8	100.0
Cabinetmakers	20,119	23,764	3,645	18.1	6,489	24.4	4,906	24.4	25.7
Carpenters and joiners	152,897	213,788	60,891	39.8	126,830	14.9	107,932	70.6	43.6
Engravers	2,916	3,214	298	10.2	1,239	13.0	1,078	37.0	72.4
Leather curriers and tanners	19,909	6,591	−13,318	−66.9	6,240	9.6	5,641	28.3	100.0
Marble and stonecutters	24,312	17,798	−6,514	−26.8	16,057	18.6	13,070	53.8	100.0
Masons	56,898	75,840	18,942	33.3	68,054	12.8	59,343	104.3	68.1
Painters, glaziers, and varnishers	65,188	82,567	17,379	26.7	33,027	12.1	29,031	44.5	40.1
Plasterers	9,218	14,368	5,150	55.9	4,926	20.9	3,896	42.3	0.0
Plumbers, gas and steam fitters	18,717	29,849	11,132	59.5	4,899	11.3	4,345	23.2	0.0
Printers, lithographers, and pressmen	23,290	22,625	−665	−2.9	7,583	10.5	6,787	29.1	100.0
Shoemakers and bootmakers	75,266	55,137	−20,129	−26.7	99,663	10.3	89,398	118.8	100.0
Saddlers and harness makers	9,718	5,174	−4,544	−46.8	5,742	4.7	5,472	56.3	100.0

of specific processes they were performing." Consequently, there were undoubtedly in 1910 somewhat less than 49,332 foreign-born white bakers, for a large proportion of those who called themselves bakers were merely performing some specific process in which they did not utilize to the full extent their experience and training as bakers in the old country. Another factor here to be taken into consideration is that the introduction of machinery, the multiplication of processes, and the division of labor have made it possible for immigrants who formerly followed other occupations to become bakers. The increase of 5,964 foreign-born white bakers from 1900–1910 may therefore be due in part to foreign-born white workers of other occupations becoming bakers.

Of foreign-born white barbers and hairdressers there were 18,836 more in 1910 than there were in 1900, an increase of 54.4 per cent. Assuming that 20.9 per cent of all barbers and hairdressers who arrived during the decade left the country again and that those remaining all engaged in their former occupations the increase would have been 55 per cent. We may infer, therefore, that the increase of foreign-born white barbers and hairdressers from 1900 to 1910 was derived largely from immigrants of the same occupation who arrived during the decade and that a small number of the new immigrants went into other occupations.

In 1910 there were 66,956 foreign-born white blacksmiths, which represents an increase of 4,006, or 6.4 per cent, over 1900. During the years 1901 to 1910, inclusive, 44,637 white immigrant blacksmiths arrived in the United States. Assuming that 9.8 per cent of these left the country again, the increase of foreign-born white blacksmiths from 1900 to 1910 would have been about 64 per cent. In other words, only about 10 per cent of the blacksmiths assumed to have remained in the United States followed their former occupations.

There were 555 foreign-born white bookbinders less in 1910 than there were in 1900, a *decrease* of 9.4 per cent. If of the 4,359 bookbinders who arrived during the decade, 94.6 per cent, or 4,124, had followed their former occupations in the United States, the *increase* over 1900 would have been about 69.8 per cent.

Foreign-born cabinetmakers in 1910, as compared with 1900, showed an increase of 3,645, or 18.1 per cent. If 75.6 per cent of all the arriving cabinetmakers had followed their former occupations, the increase would have been about 24.4 per cent. Assuming that the increase was composed entirely of immigrants who had been cabinetmakers abroad, it appears that nearly 26 per cent of the cabinetmakers supposed to have remained in this country engaged in other occupations.

In 1900 there were 152,897 foreign-born white carpenters and joiners. In 1910 the number had increased to 213,788, including 566 carpenters' apprentices and 1,605 carpenters' helpers. The figure for 1900 probably also includes apprentices and helpers. The increase in 1910 over 1900 was 60,891, or 39.8 per cent. If of the 126,830 immigrant carpenters and joiners who arrived during the decade only 85.1 per cent, or 107,932, remained and followed their former occupations, the increase in 1910 over 1900 would have been 70.6 per cent. If the increase of 60,891 in 1910 over 1900 was composed entirely of immigrants who were carpenters and joiners abroad, and if of all the incoming immigrant carpenters and joiners only 107,932 remained, then only 56.4 per cent of all those who remained engaged in their former occupation.

Of foreign-born white leather curriers and tanners there were apparently 13,318 less in 1910 than there were in 1900, a decrease of 66.9 per cent. It appears that not only have the incoming leather curriers and tanners failed to take up their former occupations, but the larger proportion of those who in 1900 followed that occupation abandoned it between 1900 and 1910. It is probable that the marked decrease in this occupation is due to the fact that the census of 1910, as has been mentioned above, aimed to record the specific operation performed rather than the trade of the informant, so that those who were returned in 1900 as tanners were returned in 1910 under such occupations as "setters-out," "sorters," "shavers and scrapers," "splitters," "scrapers," etc. It is not possible, therefore, to compare the number of foreign-born whites engaged in tanneries in 1910 and in 1900. If this comparison were possible and if it were to show an increase in the total number of foreign-born whites employed in tanneries proportionate to the number of leather curriers and tanners who arrived during the decade, it still would not follow that the increase consisted of immigrants who were leather curriers and tanners abroad, or that if the increase did consist of these workers, they were utilizing the same degree of skill and experience required abroad in the same occupations.

The foreign-born marble and stone cutters also show a marked decrease in 1910 as compared with 1900. The 16,057 immigrant marble and stone cutters who arrived during the decade 1901–1910 had apparently no effect upon the number of foreign-born white marble and stone cutters in 1910.

The foreign-born white masons show an increase of 33.3 per cent in 1910 over 1900. If 87.2 per cent of all the white immigrant masons who arrived during the decade had remained in the United States and followed their former occupations, the increase in 1910 over 1900 would have amounted to 104.3 per cent.

The 82,567 painters, glaziers, and varnishers in 1910 includes 215 apprentices and 368 helpers, as well as 723 workers designated in the census of 1910 as "painters and gilders." The increase in 1910 as compared with 1900 was 26.7 per cent. If of the 33,027 painters, glaziers, and varnishers who arrived during the decade, 29,031, or 87.9 per cent, had remained and followed their former respective occupations, the increase in 1910 over 1900 would have been 44.5 per cent.

In 1910 there were 14,368 foreign-born white plasterers. This number includes 86 apprentices and 874 helpers. In 1900 the number of foreign-born white plasterers was 9,218, an increase of 55.9 per cent. If only 79.1 per cent of the incoming immigrant plasterers had remained in the United States and worked as plasterers, the increase would have been about 42.3 per cent. These figures seem to show that immigrants who followed other occupations (or no occupations) before coming into the United States became plasterers after their arrival in this country.

In 1900 there were 18,717 plumbers, gas and steam fitters, while in 1910 there were 29,849. Of this number 628 are given in the census of 1910 as plumbers' apprentices and 2,422 as plumbers' helpers. Since the 1910 census does not classify separately gas and steam fitters, it has been assumed here that these occupations were included under plumbers. The increase in 1910 over 1900 is 59.5 per cent. If the increase had consisted only of arriving plumbers, gas and steam fitters who remained in this country and had followed their former occupations, the increase would have been only 23.2 per cent. In this case, as in the case of the plasterers, it appears that immigrants who formerly, in the old country, followed other occupations became plumbers, gas and steam fitters in this country.

In 1900 there were 23,290 printers, lithographers, and pressmen. In 1910 there were 22,625, including 17,617 workers designated as compositors and typesetters, and 225 workers designated as press hands. During the decade 7,583 printers arrived. No separate class appears in the reports of the Commissioner General of Immigration for lithographers and pressmen, or presswomen. The decrease in this occupation was 2.9 per cent, though if all the arriving immigrant printers who remained had followed their former occupations, there would have been an increase of 29.1 per cent. Apparently, then, the immigrant printers who came to the United States during the decade took up other occupations.

Of the 55,137 shoemakers reported in 1910, 44,802 were not engaged in factories, while 10,335 were designated as shoemakers (not specified) in factories. Compared with 1900, the figure for 1910 represents a decrease of 20,129, or 26.7 per cent. If the 89,398 arriving immigrant

shoemakers who remained in the United States had followed their former occupation, there would have been an increase of 118.8 per cent. It appears, then, that the old country shoemaker drifts into some other occupation after his arrival in the United States; or if he finds employment in a modern shoe factory, he specializes in some specific process requiring little or none of the skill he possessed as shoemaker at home.

It is seen that in 1910 there were 4,544 saddlers and harness makers less than there were in 1900, a decrease of 46.8 per cent. If, however, all the immigrant saddlers and harness makers who arrived in this country during the decade and remained had followed their former occupation, 1910 would show an increase of 56.3 per cent over 1900.

It will be seen that of all the occupations considered, the plasterers, and the plumbers, gas and steam fitters, are the only groups which seem to have absorbed the total net immigration. The total number of immigrants in the sixteen occupations assumed to have remained here was 424,399, while the net increase of foreign-born in all the occupations considered was 100,518. In other words, taking these sixteen skilled occupations as a whole, only 23.7 per cent of those who came to the United States and did not return took up their former occupation; 76.3 per cent scrapped their skill and experience and took up some other trade.

Table VI shows the distribution of immigrants in nine industrial and two professional occupations for the decade 1880–90. This period was chosen rather than 1890–1900 because comparable data could not be secured for the latter. It will be seen that the immigrant clergymen, physicians and surgeons, carpenters and joiners, machinists, and tailors were fully absorbed by their former occupations, while the bakers, the blacksmiths, the butchers, the shoemakers, and the masons were not.

It should be pointed out that no reliable data are available as to the number of immigrants of each occupation returning during the decade 1880–90. It has been assumed that the proportion is the same as in the period 1908–14, though this is probably an overestimate. In Table VI as in Table V it is assumed that the increase in the respective occupations during the decade came entirely from new immigrants who had followed the same occupations abroad. On this basis it appears that the proportion of immigrants who abandoned their former occupations varied from zero in several cases to 65.7 per cent in the case of the shoemakers. The large increase of foreign-born machinists seems to indicate that the census returns included a considerable number of helpers and apprentices and perhaps other employees in machine

TABLE VI

FOREIGN-BORN, BOTH SEXES, ENGAGED IN SPECIFIED OCCUPATIONS IN 1880 AND IN 1890, INCREASE 1880 TO 1890, THE NUMBER OF IMMIGRANTS ARRIVING AND THE ESTIMATED NUMBER REMAINING IN THE UNITED STATES DURING THE DECADE 1880–1890, AND THE PER CENT ESTIMATED TO HAVE ABANDONED THEIR FORMER OCCUPATIONS

(From U. S. Census 1880, vol. 10, pp. 744–51; U. S. Census 1900; special report on occupations pp. clxxxvii–clxxxix; Report of Commissioner General of Immigration 1892, pp. 44–50.)

Occupation	Number engaged		Increase 1880–1890		Number arriving 1881–1890	Per cent departing 1908–1914	Number remaining from those who arrived 1881–1890	Number remaining 1881–1890, expressed as per cent of total enumerated in 1880	Per cent of immigrants who abandoned their former occupations 1881–1890
	1880	1890	Number	Per cent					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Total of eleven occupations	540,470	830,584	290,114	....	279,485	25.1	209,334	38.7	0.0
Clergymen	12,731	18,387	5,656	44.4	3,534	27.2	2,573	20.2	0.0
Physicians and surgeons	8,579	11,185	2,606	30.3	2,253	31.4	1,546	18.0	0.0
Bakers	23,201	35,117	11,916	51.3	18,674	15.2	15,836	68.3	24.8
Blacksmiths	47,130	64,851	17,721	37.6	27,405	9.8	24,719	53.4	28.3
Butchers	29,230	38,993	9,763	33.4	17,799	12.6	15,556	53.2	37.2
Carpenters and joiners	85,691	160,006	74,315	86.7	64,468	14.9	54,862	64.0	0.0
Miners	126,235	195,439	69,204	54.8	49,091	117.8	.....	....	....
Shoemakers	69,388	78,150	8,762	12.6	28,481	10.3	25,547	36.8	65.7
Machinists	30,458	57,457	26,999	88.6	3,634	30.2	2,537	8.3	0.0
Masons	36,246	60,867	24,621	67.9	30,418	12.8	26,524	73.2	7.2
Tailors	71,581	110,132	38,551	53.8	33,728	11.1	29,984	41.9	0.0

shops who were not skilled machinists. It is evident that a large number of immigrants previously engaged in other occupations during the decade 1880–90 entered such professions and occupations as clergymen, physicians and surgeons, carpenters and joiners, miners, machinists, and tailors.

The Immigration Commission in its study of immigrants in industry made inquiry as to the occupation and experience of the foreign-born before coming to this country. The results of this inquiry are presented in Table VII. The most striking fact revealed is the very small per cent of employees in certain industries who had been employed in the same occupation abroad. The proportion of farm laborers is high, but perhaps not so high as might be expected.

TABLE VII

EMPLOYEES AND HOUSEHOLDS STUDIED BY THE IMMIGRATION COMMISSION IN FIFTEEN PRINCIPAL INDUSTRIES, THE PROPORTION OF THE EMPLOYEES WHO WERE FOREIGN-BORN AND THE PERCENTAGES OF THE FOREIGN-BORN WHO HAD BEEN EMPLOYED AT THE SAME OCCUPATIONS OR HAD BEEN FARMERS OR FARM LABORERS ABROAD, BY INDUSTRY AND SEX

(From Abstracts of Reports of the Immigration Commission, vol. 1, pp. 297–313)

Industry	Number of employees from whom detailed information was secured	Number of households studied	Per cent of employees foreign-born	Per cent of foreign-born employees at same occupation abroad			Per cent of foreign-born employees who were farmers or farm laborers abroad		
				Male	Female	Both sexes	Male	Female	Both sexes
Iron and steel	86,089	2,456	57.7	....	....	8.6	....	....	64.4
Slaughtering and meat packing	43,502	1,039	60.7	5.1	0.5	....	58.4	82.8	....
Bituminous coal mining	88,368	2,371	61.9	....	....	20.7	....	....	58.0
Woolen and worsted mfg.	23,388	440	61.9	22.1	41.9	....	42.4	34.5	....
Silk goods mfg. and dyeing	12,994	272	34.3	73.9	76.1	....	6.5	7.5	....
Cotton goods mfg.	66,800	1,061	68.7	15.8	34.5	....	56.2	50.7	....
Clothing mfg.	19,502	906	72.2	62.5	75.6	....	....	....	....
Boots and shoes	19,946	710	27.3	....	....	41.4	....	....	29.2
Furniture mfg.	4,295	338	59.1	....	....	10.4	....	....	49.3
Collar, cuff and shirt mfg.	1,508	264	13.4	....	3.6	....	....	35.7	....
Leather, tanning, currying and finishing	12,839	362	67.0	....	....	6.0	....	....	58.1
Glove mfg.	908	262	33.5	60.9	14.4	....	21.2	5.5	....
Oil refining	6,123	525	66.7	0.0	0.0	....	....	....	60.8
Sugar refining	5,826	194	85.3	....	....	0.6	....	....	60.8
Cigars and tobacco	35,564	127	32.6	55.3	29.2	....	23.3	51.6	....

CONCLUSION

Although the lack of uniformity of occupational designations and the absence of satisfactory information with regard to the number of departing immigrants and their occupations make it impossible to take the above figures literally, the available statistics do, nevertheless, seem to show definitely that on the whole neither the immigrant agricultural workers nor the immigrant skilled workers follow their

former occupations to a large extent after coming to the United States. The fact that many occupations show an increase of foreign-born entirely disproportionate to the number of immigrants of the same occupation who came into the country, indicates that immigrants choose an occupation without much regard to their previous training and experience. This fact also suggests that special skill and training are not generally required of immigrants. It is probable that in occupations showing a disproportionate increase of foreign-born the immigrants in reality perform specific processes, though still parading under the general trade name. The large increase of tailors from 1880 to 1890, for example, shows that many immigrants learned the tailoring trade in this country in a much shorter time than it would have taken them to learn the same trade abroad. The division of labor and the multiplication of machinery referred to above have made it possible for many trades to employ immigrants regardless of their former training and experience and have probably, on the other hand, compelled many immigrants to abandon their previous occupations.

It has not been assumed in this study that it is to the best interests of the foreign-born or of society for the immigrant to engage in this country in the same occupation he followed abroad. Many immigrants leave the old country with the hope of finding in America an occupation more suitable for their health or tastes; others for the purpose of finding more lucrative employment.

If each skilled occupation in this country pays better wages than the corresponding occupation in the old country, it becomes necessary to regulate the influx of immigrants in these occupations in order to keep up the better wage. If this regulation is to be accomplished by those already in the trade, it probably requires a closed shop to exclude the new immigrant, thus compelling him to seek employment in another trade.

It is probable, by reason of our specialized industrial processes, that the number of skilled immigrant workers coming into this country is in excess of the number needed. From a social point of view, therefore, it need not follow that the skilled immigrant workers should engage in occupations which would enable them to utilize their acquired trade skill and training, or that skilled immigrants are more desirable than unskilled.

The fact, for instance, that apparently none of the 89,398 shoemakers who arrived during 1900–1910 and are assumed to have remained in the United States, engaged in their former occupations may be to the best interests of the shoemakers themselves as well as to the best interests of the country. The figures (Table V) show that there

was a decrease of foreign-born boot and shoemakers in 1910 as compared with 1900, amounting to about 27 per cent. If all the new shoemakers had found employment as shoemakers, there would have been an increase of more than one hundred per cent instead of the decrease of 27 per cent. The influx of shoemakers during that decade was obviously disproportionate to the needs of that particular industry, but probably was not in excess of the needs of all industries.

NOTES

A USE FOR TRIGONOMETRIC TABLES IN CORRELATION

By Holbrook Working, *University of Minnesota*

In any problem in partial, or multiple, correlation, it is necessary to find the value of $\sqrt{1-r^2}$ or Log $\sqrt{1-r^2}$ for several different values of r. The same quantity also appears in the formula for the standard error of an equation of regression, $s_y = \sigma_y\sqrt{1-r^2}$.*

Where r varies between the limits $+1$ and -1, as is always the case in these formulae, there is an interesting and useful trigonometric relation between r and $\sqrt{1-r^2}$. Any one familiar with analytical geometry will recognize the equation $y=\sqrt{1-r^2}$ as the equation of a circle. There is also a simple geometrical proof of the relation. Draw the arc YAX with radius $OA=1$. Construct AX_1 perpendicular to OX_1, giving the right triangle OAX_1. By the familiar relation that the square of the hypotenuse equals the sum of the squares of the other two sides, we have $(AX_1)^2=(OA)^2-(OX_1)^2$. But OA being taken equal to unity and $OX_1=r$, we have $AX_1=\sqrt{1-r^2}$.

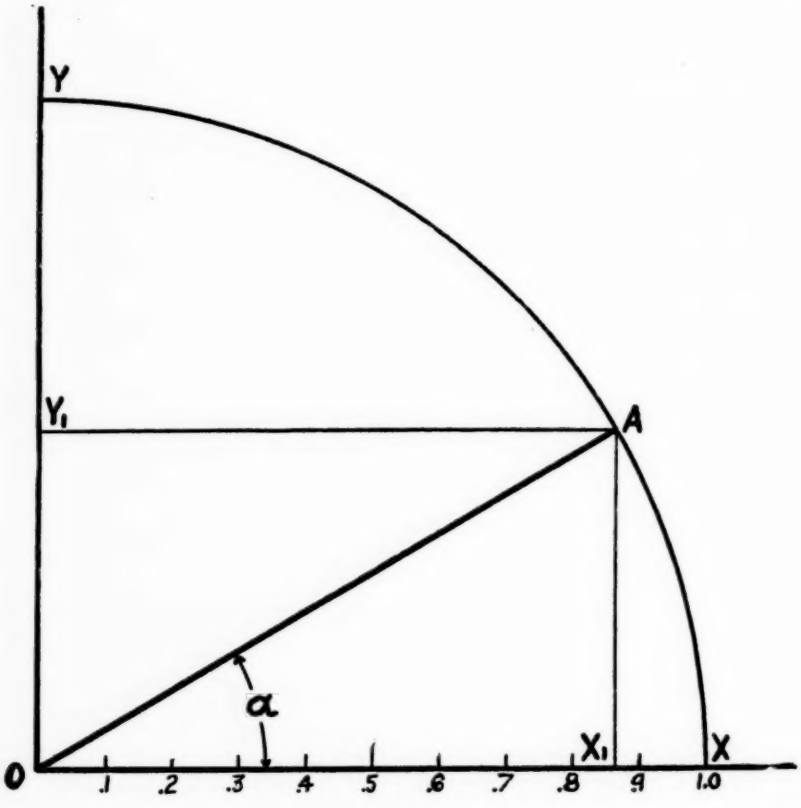
Now it is immediately apparent that AX_1 is the sine of angle a and that OX_1 is the cosine of angle a. It follows that if we find the angle whose cosine is r, we need but take the sine of that angle to get $\sqrt{1-r^2}$, or the Log of the sine to get Log $\sqrt{1-r^2}$. The reverse is also true. Briefly:

If $r=\cos a$, $\sin a=\sqrt{1-r^2}$, Log $\sin a=$ Log $\sqrt{1-r^2}$.

If $r=\sin a$, $\cos a=\sqrt{1-r^2}$, Log $\cos a=$ Log $\sqrt{1-r^2}$.

Knowing these relations, the value of $\sqrt{1-r^2}$ or of Log $\sqrt{1-r^2}$ may quickly be found in any table of the trigonometric functions. The accuracy with which these values may be found is very fair. Without interpolation a table giving the trigonometric function for each minute of arc gives an accuracy sufficient for the purpose of ordinary statistical investigation. For example, take $r=0.20$. The angle (of even minutes) whose sine is nearest 0.20 is the angle 11° 32′. Since fractions of minutes are neglected, the error in the angle taken may be ½ minute, but not more than ½ minute. The cosine of angle 11° 32′ is 0.97981. A difference of ½ minute in the angle will change the cosine by 0.00003.

* The notation is that of Yule. Cf. Yule, *Introduction to the Theory of Statistics*, 5th Edition, p. 177.



This is the maximum possible error in the determination of $\sqrt{1-r^2}$ when $r=0.20$. For higher values of r the error is greater. If $r=0.99$ the maximum possible error in the determination of $\sqrt{1-r^2}$ is 0.00071. For $r=0.99$, again, the maximum possible error in the determination of Log $\sqrt{1-r^2}$ is 0.00044.

Therefore we may conclude that when a table giving the trigonometric functions for each minute of arc is used without interpolation, the value obtained for $\sqrt{1-r^2}$ or for Log $\sqrt{1-r^2}$ is accurate to at least three decimals. By interpolation, accuracy to at least four decimals may be obtained.

The accompanying table was prepared by the method outlined above, but using tables giving the functions to eight decimals for each sexagesimal second of arc.* In the table values of Log $\sqrt{1-r^2}$

* The tables used were E. Gifford, *Natural Sines*, Manchester, 1914, and Bauschinger and Peters, *Logarithmic-Trigonometrical Tables*, Leipzig, 1911.

for the higher values of r may be in error as much as 0.000,004, but not more. The accuracy of the values given for $\sqrt{1-r^2}$ is slightly better.

VALUES OF $\sqrt{1-r^2}$ AND $\text{Log}\sqrt{1-r^2}$ TO SIX DECIMAL PLACES FOR VALUES OF r FROM .00 TO .99 FOR EACH CHANGE OF .01 IN r.*

r (sin α(	$\sqrt{1-r^2}$ (cos α)	Log $\sqrt{1-r^2}$ (Log cos α)	r (sin α)	$\sqrt{1-r^2}$ (cos α)	Log $\sqrt{1-r^2}$ (Log cos α)
.00	1.000 000	0.000 000	.50	.866 025	9.937 531
.01	.999 950	9.999 978	.51	.860 174	9.934 586
.02	.999 800	913	.52	.854 167	1 543
.03	.999 550	804	.53	.847 997	9.928 394
.04	.999 200	652	.54	.841 665	5 139
.05	.998 749	456	.55	.835 165	1 772
.06	.998 198	217	.56	.828 492	9.918 288
.07	.997 547	9.998 933	.57	.821 644	4 684
.08	.996 795	606	.58	.814 616	0 953
.09	.995 942	234	.59	.807 404	9.907 091
.10	.994 983	9.997 818	.60	.799 999	3 089
.11	.993 932	357	.61	.792 402	9.898 946
.12	.992 774	6 850	.62	.784 602	4 649
.13	.991 551	299	.63	.776 596	0 195
.14	.990 152	5 702	.64	.768 374	9.885 572
.15	.988 686	058	.65	.759 933	0 776
.16	.987 117	4 369	.66	.751 264	9.875 793
.17	.985 444	3 632	.67	.742 363	0 616
.18	.983 667	2 848	.68	.733 212	9.865 230
.19	.981 784	2 016	.69	.723 807	9.859 623
.20	.979 796	1 136	.70	.714 144	9.853 786
.21	.977 702	0 206	.71	.704 201	9.847 696
.22	.975 499	9.989 227	.72	.693 975	9.841 344
.23	.973 190	8 198	.73	.683 447	9.834 705
.24	.970 772	7 117	.74	.672 507	9.827 762
.25	.968 246	6 986	.75	.661 439	9.820 490
.26	.965 609	4 801	.76	.649 923	9.812 862
.27	.962 861	3 564	.77	.638 044	9.804 851
.28	.960 000	2 271	.78	.625 780	9.796 422
.29	.957 026	0 924	.79	.613 106	9.787 536
.30	.953 940	9.979 521	.80	.600 001	9.778 152
.31	.950 737	8 060	.81	.586 431	9.768 217
.32	.947 417	6 541	.82	.572 365	9.757 673
.33	.943 980	4 963	.83	.557 765	9.746 451
.34	.940 425	3 324	.84	.542 584	9.734 467
.35	.936 750	1 624	.85	.526 783	9.721 632
.36	.932 952	9.969 859	.86	.510 293	9.707 819
.37	.929 032	8 031	.87	.493 052	9.693 893
.38	.924 987	6 136	.88	.474 972	9.676 668
.39	.920 815	4 172	.89	.455 959	9.659 926
.40	.916 516	2 140	.90	.435 890	9.639 377
.41	.912 086	0 036	.91	.414 609	9.617 639
.42	.907 523	9.957 858	.92	.391 918	9.593 195
.43	.902 829	5 606	.93	.367 561	9.565 329
.44	.897 998	3 275	.94	.341 173	9.533 974
.45	.893 029	0 866	.95	.312 252	9.494 505
.46	.887 918	9.948 373	.96	.280 001	9.447 160
.47	.882 667	5 797	.97	.243 107	9.386 798
.48	.877 270	3 133	.98	.198 993	9.299 847
.49	.871 723	0 378	.99	.141 068	9.149 429

* Professor Bruce D. Mudgett has called my attention to a similar table, carried to a smaller number of decimals, appearing in a Bulletin of the University of Texas: *Tables to Facilitate the Calculation of Partial Coefficients of Correlation and Regression Equations*, by Truman Lee Kelley.

A study of the accompanying table results in some interesting conclusions regarding the accuracy with which r ought to be obtained in calculating the original correlation. The following relations are found:

For values of r up to

0.10 0.45 0.71 0.89 0.95 0.97

a change of 0.01 in r changes $\sqrt{1-r^2}$ less than:

0.001 0.005 0.01 0.02 0.03 0.04.

From the above it appears that when r does not exceed 0.71, $\sqrt{1-r^2}$ may be found to at least the same degree of accuracy as r.* For values of r over 0.71, $\sqrt{1-r^2}$ is not accurate to as many decimals as r, but is at least accurate to one decimal less than r.

It follows that in order to have $\sqrt{1-r^2}$ accurate to two decimals, values of r above 0.71 should be calculated to three decimals.

In order to have $\sqrt{1-r^2}$ accurate to *three* decimals, the value of r must be calculated as follows:

r not over 0.10	2 decimals suffice
r not over 0.71	3 decimals suffice
r over 0.71	4 decimals suffice

Taken in itself, the circumstance that in order to get $\sqrt{1-r^2}$ accurate to three decimals, r must be calculated to at least four decimals if it be larger than 0.71, does not prove that r should be calculated to four decimals. It has generally been considered that r should not be calculated beyond two or, at the most, three decimals, further accuracy being fictitious. If this assumption be well founded, it follows that the value of $\sqrt{1-r^2}$ may not be accurately found beyond one or, at the most, two decimals, if r be over 0.71. Which is the correct conclusion: that where r is over 0.71, its value should be calculated to a larger number of places than for lower values; or that where r is over 0.71, $\sqrt{1-r^2}$ cannot be found accurately to as many places as for lower values of r?

The problem may be approached by a consideration of the properties of the standard error ($\sigma_y \sqrt{1-r^2}$), which measures the dispersion of the observations about the line of regression. Given the equation of regression, the standard error may be calculated quite independently of the coefficient of correlation by finding the deviation of each observation from the line of regression and calculating the root-mean-square of these deviations. The writer sees no reason for considering that the standard error can be determined with any greater accuracy where the correlation is low than where the correlation is high. But if the standard error, which is $\sigma_y \sqrt{1-r^2}$, may be determined with as great accuracy for high values of r as for low values, it follows, since σ_y is not affected by the closeness of the correlation, that $\sqrt{1-r^2}$ may

* The accuracy referred to is, of course, not a question of the method used, but is inherent in the relation of r to $\sqrt{1-r^2}$.

be determined with as great accuracy for high values of r as for low values. If this be true, it follows further that for high values of r its value should be calculated to a larger number of decimals than for low values.

Since, as noted above, a table giving the sines and cosines for each minute of arc gives $\sqrt{1-r^2}$ accurate to at least three decimal places without interpolation, such a table used without interpolation is adequate where r is calculated to the number of decimals indicated in the preceding table, that is, where r is calculated so as to give $\sqrt{1-r^2}$ accurate to three decimals.* Seldom, if ever, is it desirable to calculate the value of r to a larger number of decimals; the apparent greater accuracy would be fictitious.

THE GRAPHIC REPRESENTATION OF A FREQUENCY DISTRIBUTION

By Robert E. Chaddock, *Columbia University*

In the accompanying diagrams and the explanatory text the purpose is two-fold: (1) to present a method of exposition by which the student of elementary statistical methods will be enabled to comprehend more clearly the nature of the frequency distribution and the assumptions underlying grouped data; (2) to correct or clarify the rules for locating the median and quartiles which appear in some of the widely used elementary text-books. The student is told to add one to the number of cases and to divide by two $\frac{(n+1)}{2}$ in order to locate the mid-item in the array, the value of which is the median average. But if this rule is followed, slightly different values are secured for the median, depending on whether the calculation is made from the lower toward the higher values in the array, or from the higher toward the lower, whereas either procedure should yield exactly the same result. The reason is clear. One extra item has been inserted. After all, what we seek is not a case but a value in an array of items arranged in a regularly ascending or descending order of values, which are assumed to be evenly distributed over the scale within a given class interval so that progress in one direction shows constantly increasing values, and in the other direction constantly decreasing values.

* For this purpose the most convenient tables the writer has seen are in G. W. Jones: *Logarithmic Tables*. London: Macmillan, 1898. These tables give the natural and logarithmic functions in parallel columns for each minute of arc.

DIAGRAM I.

DISTRIBUTION OF PIECE-RATE EARNINGS IN 50 CENT GROUPS

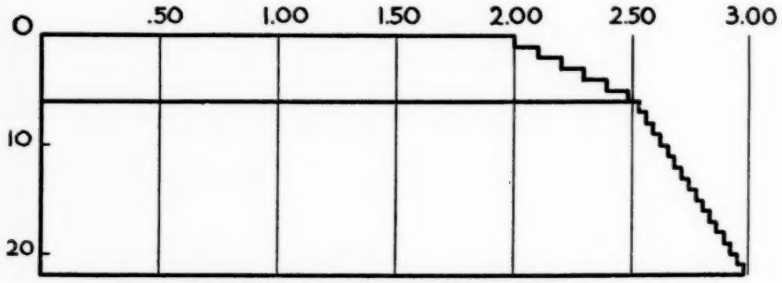


Fig. 1 SHOWING INDIVIDUAL VARIATIONS

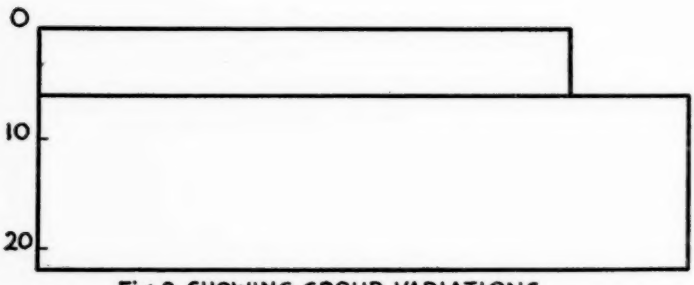


Fig. 2 SHOWING GROUP VARIATIONS

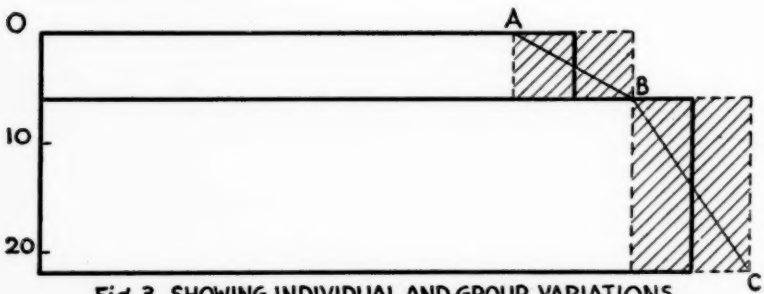


Fig. 3 SHOWING INDIVIDUAL AND GROUP VARIATIONS

The data in the illustration, presented in the form of a frequency table in Diagram II, are distributed with a fair degree of regularity because piece-rate earnings are used in 50 cent groups.* The amount of the wage is shown on the horizontal scale, and the number of wage earners on the vertical scale. Figure 1 in Diagram I shows the first two groups at the lower values on the scale distributed by individual wage earners as well as by groups, *on the assumption of even distribution* of 6 cases in the first group and 16 cases in the second group. A rectangular step on the horizontal scale and a space on the vertical scale are shown for each wage item. The distance between the limiting lines on the vertical scale shows the number of items in each group, 6 and 16 respectively.

Figure 2 shows the same facts, but the 6 items in the lower group are now given the same value on the horizontal scale concentrated at the middle of the 50 cent range, at $2.25, as would be given in fact in calculating the arithmetic average of the entire array shown in Diagram II. Likewise, the 16 items of the second group are concentrated at $2.75.

Figure 3 combines the facts of both figures 1 and 2. The cross-hatched areas show two things: (a) the range along the horizontal scale over which the items of the group are actually distributed from the smallest value to the largest, and (b) the number of items in each group, measured on the vertical scale. The diagonal *A-B* shows the increasing values in that group of 6 items, from the lowest values at *A* to the greatest at *B*, which was indicated by the steps in figure 1. The diagonal *B-C* shows the same for the 16 items of the second group, the steps in each case having been smoothed by the straight lines *A-B-C*.

Here it may be objected that values are not usually distributed over the range of class intervals so regularly as indicated in figures 1 and 3. This situation is admitted. They may be more closely grouped at the lower or upper limits of the given interval or over the upper half of the interval as compared with the lower half. But this very objection merely emphasizes the importance of the choice of the width of the interval for classifying the items, and the importance of locating experimentally the lower and upper limits of the interval. If natural massing points of the items can be discovered, these should constitute the mid-values of the class interval, shown in figure 2, as far as possible, at which all the values of the group are concentrated for practical purposes in calculating the arithmetic average. Thus, less error will be

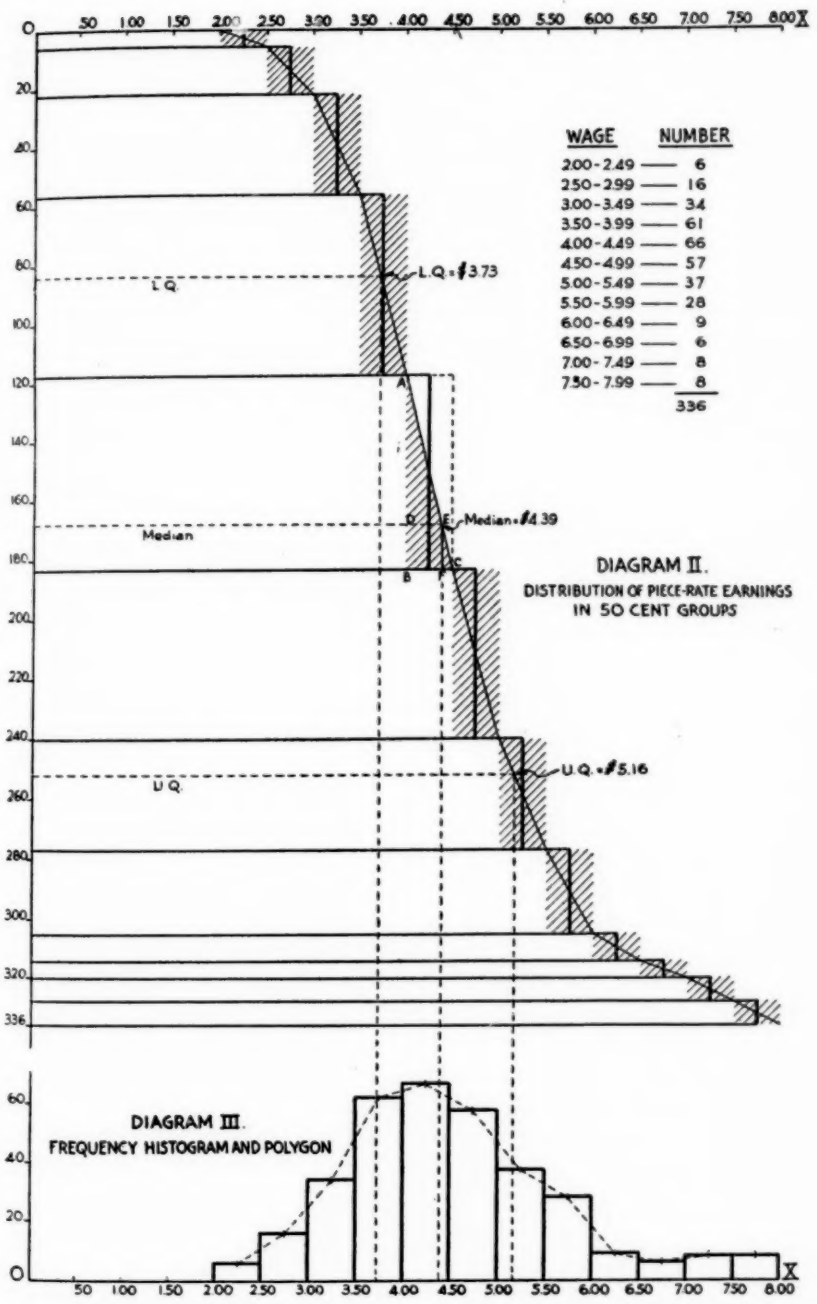
* The unit in which wages are paid is not capable of indefinite subdivision as in the case of measurement of height, weight, or time. This fact may affect the distribution over the class interval and the massing of cases at certain values on the scale. This is especially true in time wages, but we are using piece-rate earnings where the smallest subdivision of the unit of payment is one cent. Therefore, within any class interval of 50 cents there are many subdivisions possible, and the assumption for such data as height and weight may be made and the same methods of treatment may be employed.

introduced in calculating the arithmetic average from grouped data as compared with the true average calculated from the individual items without grouping. In most data some classification and grouping of items is essential for ease and clearness in handling.

In Diagram II the procedure of figure 3 in Diagram I has been carried out for each group in the entire array of 336 wage items. The limiting lines on the vertical scale show the number of cases in each group, as does also the height of the cross-hatched areas. The total height of cross-hatched areas indicates the total of the items, 336. In each group the width of the cross-hatched area is the same, measuring on the horizontal scale the range of variation for that group, the 50 cent interval. Now, if we project or let fall all the cross-hatched areas upon a common base line *O-X*, shown in Diagram III, in the exact relation and size on the horizontal and vertical scales in which they are found in Diagram II, the result will be a *surface of frequency*, sometimes called a *histogram* or *column diagram*. The height of each rectangular area above the base line represents the proper number of cases in that group, and the total area represents the total items. The total cross-hatched area in Diagram II is identical with the total area of Diagram III. By the dotted line connecting the mid-points of the tops of the columns, in Diagram III, the histogram may be readily converted into the *frequency polygon*, and by appropriate methods this may be further smoothed into a *frequency curve*, showing the distribution of values around a central tendency. This procedure is fundamental in statistical analysis.

THE EXACT LOCATION OF THE MEDIAN VALUE IN GROUPED DATA

Using Diagram II and noting that the number of items is even (336), we must locate a value on the horizontal scale, such that there will be as many value items above as below. Half the items would be $\frac{336}{2} =$ 168 items. If we count down on the vertical scale through the first four groups there will be 117 items. This brings us to the wage item of \$4.00 on the horizontal scale. If we count in addition all of the fifth group, which has 66 items in it, there will be 183 items in the first five groups, up to \$4.50 on the horizontal scale. But we wish a value that will show only 168 items below and the same above. Therefore, the median value must lie somewhere in the fifth group between \$4.00 and less than \$4.50. This group of 66 items is distributed evenly, by assumption, over the 50 cent interval. The lowest value in this group is at \$4.00 and the highest just under \$4.50, as shown by the diagonal *A-C* and the distance *B-C* in Diagram II. How many of the 66 items do we need,


WAGE NUMBER
2.00-2.49 — 6
2.50-2.99 — 16
3.00-3.49 — 34
3.50-3.99 — 61
4.00-4.49 — 66
4.50-4.99 — 57
5.00-5.49 — 37
5.50-5.99 — 28
6.00-6.49 — 9
6.50-6.99 — 6
7.00-7.49 — 8
7.50-7.99 — 8
336
L.Q.
L.Q. = $3.73
Median
Median = $4.39
U.Q.
U.Q. = $5.16
DIAGRAM II.
DISTRIBUTION OF PIECE-RATE EARNINGS IN 50 CENT GROUPS
DIAGRAM III.
FREQUENCY HISTOGRAM AND POLYGON

added to the 117 of the first four groups, to complete the 168 items which must lie below the median value of the entire array? There are needed 168 − 117 = 51 items to complete the 168 within the fifth group, as shown by the distance *A-D* measured on the vertical scale. Now these items in this group are increasing in size as we count from *A* toward *D*, as shown along the diagonal *A-C*. Beginning with \$4.00 at *A*, the lower limit of the group, how much greater will the value be when we have counted 51 items along the vertical scale toward *B*? *A-B* represents 66 items, *B-C* represents 50 cents, and *A-D* represents 51 items. What will be the value range represented by *D-E*? The triangles *ABC* and *ADE* are similar. Therefore, the proportion *AB*:*BC*::*AD*:*DE* will be true. The value to be found is *DE*. Substituting known values, we have 66:50 cents::51:DE, or $66DE = 2550$, and $DE = 39$ cents. Adding 39 cents to \$4.00, the lower limit of the group, we have \$4.39, the median value.

Or, reasoning in terms of proportions, on the assumption of even distribution, it follows that if 66 items are distributed over the entire range of 50 cents (*B-C*), in the fifth group, 51 items which must be added in order to have 168 items below the median value, will range over $\frac{51}{66}$ of 50 cents, or 39 cents (*D-E*), which amount must be added to the lowest wage item of the group, \$4.00, in order to arrive at the median value at *E*.

Exactly the same value for the median will be secured if we begin at the bottom of Diagram II, the highest wage values, and count upward on the vertical scale of the diagram. As we proceed the wage items grow progressively smaller in each group. At the upper limit of the group of 66 items (*C*), where the value is \$4.50, we have counted 153 items. It requires 15 more items in the group of 66 to fulfil the condition that 168 wage items must be greater than the median value. In the previous calculation we counted into this same group 51 items from the lower limit in order to arrive at the median. Now, we count 15 items from the upper limit to arrive at exactly the same point. $AD = 51$, *DB* or $EF = 15$, and $AD + EF = 66$, the entire group. The wage values are now decreasing along the diagonal *C* to *E* where the median lies. The value to be subtracted from \$4.50 may be secured as before by a proportion *AB*:*BC*::*EF*:*FC*, or 66:50 cents::15:*FC*, which gives 11 cents for the value of *FC*; \$4.50 − .11 = \$4.39; the median value, or if 66 items range over 50 cents (*B-C*), 15 items will range over $\frac{15}{66}$ of 50 cents, or 11 cents (*F-C*).

The quartiles may be secured by dividing the items by 4 and proceeding in exactly the same manner to locate the value in the array which has $\frac{1}{4}$ of the items below and $\frac{3}{4}$ of the items above it, and also the value in the array which has $\frac{3}{4}$ of the items below and $\frac{1}{4}$ above it.

Let a perpendicular fall from *E* in Diagram II upon the base line *O-X* and it will divide the area of Diagram III into two equal parts, according to the definition of the median. Like perpendicular lines, let fall from the quartile positions of Diagram II, will divide Diagram III into four equal areas.

The deciles and percentiles may be located in a similar manner.

OBSERVATIONS ON THE $\frac{n+1}{2}$ PROCEDURE FOR LOCATING THE MEDIAN

This rule is an expedient for locating the mid-case if the numbers in the array are odd, and a position between the two mid-cases if the number of cases is even. It is not a case we seek but a value. Changing values through the array are associated with movement in one direction or the other on the assumption of an even distribution over the class interval. If one case is added it will not be possible to secure the same value for the median when calculated from both the lower and upper limits of the array because the extra case introduces a slight duplication.

It is not necessary to introduce this contradiction in the procedure. In Diagram I, figure 1, a rectangular area was shown for each item, as well as for the group. A space on the vertical scale represented each item. If the number of spaces on the vertical scale, corresponding to the number of items, is even, as in our illustration in Diagram II, we count through $\frac{1}{2}$ of the spaces on the vertical scale, which brings us to a point on the line separating the two spaces representing the two mid-items. On the horizontal scale this represents a value interpolated, or the average value of the two mid-items. If the number of items is odd, in like manner we locate the mid-point on the vertical scale and find it in the middle of the space representing the mid-item of the array, with an equal number of spaces above and below it. On the horizontal scale this represents the value of the mid-item, with an equal number of items above and below it. This may be shown by supposing that the illustration in Diagram II has 337 items. Therefore, when calculating the median, quartiles, etc., for grouped data, we should use the actual number of items instead of adding one.

SHORT TERM BORROWING IN BRITISH FINANCE, 1914–1920

The accompanying charts showing the quarterly receipts of the British Treasury during the past six and a half years make it possible to compare in the case of British public finance the relative importance of different methods of raising funds during this period. The statistics upon which the charts are based have been collected by the Division of Analysis and Research of the Federal Reserve Board in connection with studies which are being made of foreign banking and finance.

The charts clearly indicate the outstanding importance of the short term Treasury bill as a source of public funds. As in the case of the United States Treasury, the British Treasury depends upon the sale of short term bills to provide cash in anticipation of receipts from taxes and longer term loans. Before the war the same method was used, but on a scale in no sense comparable with that of the war period. Treasury bills outstanding before the war did not exceed £20,000,000, whereas in recent years they have frequently exceeded the billion pound mark.

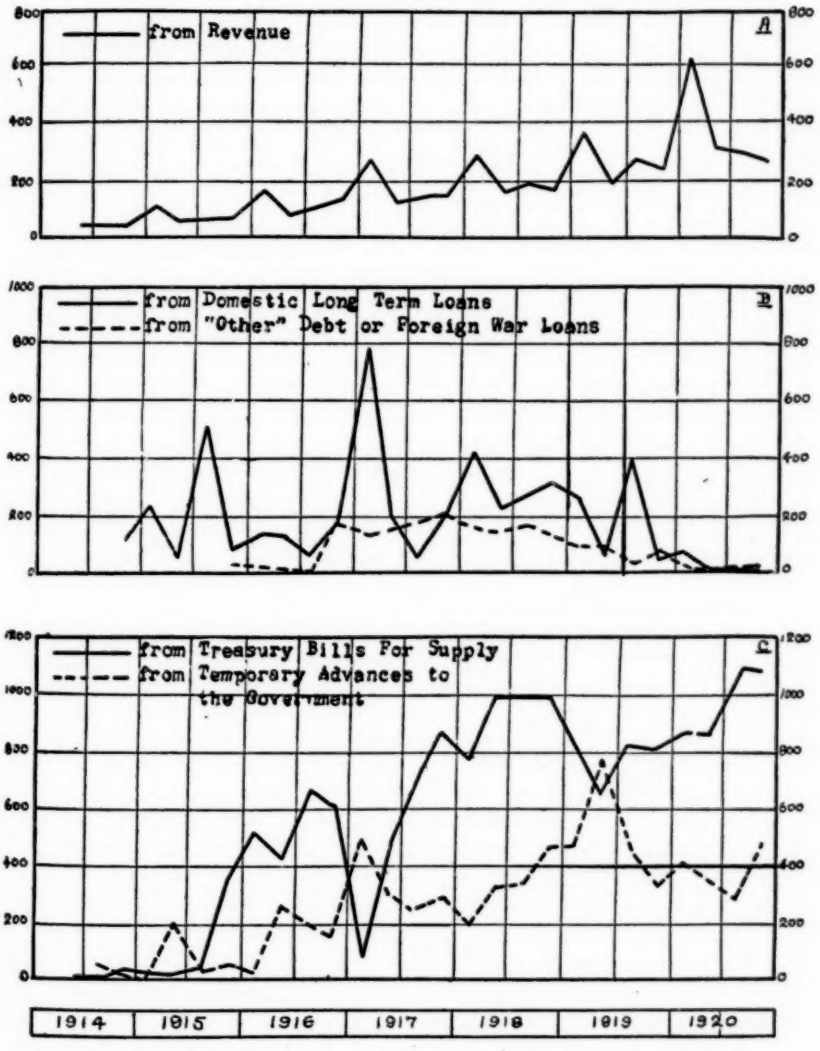
In the United States the Treasury depends entirely upon the sale of certificates of indebtedness to supply it with temporary funds. In England, on the other hand, as in France, a supplementary method is used in case Treasury bill sales are insufficient to provide the necessary cash. In France the Treasury under these circumstances obtains advances from the Bank of France. In England the Treasury applies to the Bank of England or to other public departments. The sale of Treasury bills, however, is the preferred method, since they furnish a form of investment which may be taken up by real savings, while advances from the Bank of England consist of credits created on the books of the Bank and obviously make directly for inflation. Advances from "other public departments," it is understood, are mainly drawn from the currency notes fund.

Charts B and C show that during the first year of the war, two large war loans were placed in England, providing sufficient funds in addition to those from revenue to make unnecessary the sale of Treasury bills on a large scale. During the following period, on the other hand—covering late 1915 and 1916—no long term loans were placed, and the Treasury depended very largely upon the returns from the sale of Treasury bills to provide current funds. In January 1917, the third big war loan was placed, providing for the funding of £131,000,000 of Treasury bills then outstanding; but immediately following the floating of the loan it was necessary again to turn to Treasury bills as a source of current funds. Between this time and the end of 1918, more and more Treasury bills were issued; Treasury receipts from this source amounted to £5,762,000,000, while receipts from long term loans amounted to £1,216,000,000, and from taxes £1,684,000,000. In 1919 only a small number of Treasury bills were funded in the loan floated in June, and since that time no long term loans have been placed.

The importance of the temporary advances by the Bank of England and other public departments is clearly shown in Chart C. Ever since early in 1916 it has been necessary for the Treasury to obtain such advances when receipts from Treasury bills were insufficient or when large numbers of them matured. In the first quarter of 1917 and again in 1919, these advances on ways and means actually exceeded receipts from Treasury bills. In both cases this occurred during a period when the sale of Treasury bills had been suspended (at the time of the floating of a long term loan) and when a considerable number of bills were being funded.

Although the total debt of England was somewhat reduced during 1920 (owing to repayment of certain foreign loans), the floating debt was increased by £59,000,000. The entire increase occurred in ways and means advances. How to handle this large floating debt—which amounted to £1,408,000,000 at the end of 1920—is a problem

QUARTERLY RECEIPTS BY THE BRITISH TREASURY, 1914-1920
In millions of pounds sterling



which is considerably under discussion in England. It may be possible to fund in part the Treasury bills outstanding. To pay them off even gradually will involve a heavy rate of taxation for years to come. The elimination of borrowing on ways and means is of even more pressing importance, however, since these advances result in an increase in the balances of the joint stock banks at the Bank of England.

KATHARINE SNODGRASS

SUMMARY AND GROUP CORRELATION

The possibility of finding the arithmetic mean of several groups separately, and combining the data in order to find the arithmetic mean of the combined groups, is well known and is frequently used to save the labor of additional computations. It may not be generally known that an analagous process is available in the case of the coefficient of correlation.

Let x and y be the deviations, and A and B the averages of two series, the original values being expressed by X and Y.

Then
$$\begin{aligned}\Sigma xy &= \Sigma(X-A)(Y-B)\\ &= \Sigma XY + nAB - A\Sigma Y - B\Sigma X\\ &= \Sigma XY + \frac{n(\Sigma X)(\Sigma Y)}{n\ n} - \frac{(\Sigma X)(\Sigma Y)}{n} - \frac{(\Sigma X)(\Sigma Y)}{n}\\ &= \Sigma XY - \frac{(\Sigma X)(\Sigma Y)}{n}\end{aligned}$$

$$\begin{aligned}\Sigma x^2 &= \Sigma(X-A)^2\\ &= \Sigma X^2 - 2A\Sigma X + nA^2\\ &= \Sigma X^2 - 2\frac{(\Sigma X)(\Sigma X)}{n} + n\frac{(\Sigma X)^2}{n^2}\\ &= \Sigma X^2 - \frac{(\Sigma X)^2}{n}\end{aligned}$$

and, similarly, $\Sigma y^2 = \Sigma Y^2 - \dfrac{(\Sigma Y^2)}{n}$.

Hence
$$\begin{aligned}r = \frac{\Sigma xy}{n\sigma_x\sigma_y} &= \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}}\\ &= \frac{\Sigma XY - \dfrac{(\Sigma X)(\Sigma Y)}{n}}{\sqrt{\left[\Sigma X^2 - \dfrac{(\Sigma X)^2}{n}\right]\left[\Sigma Y^2 - \dfrac{(\Sigma Y)^2}{n}\right]}}\\ &= \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{[n\Sigma X^2 - (\Sigma X)^2][n\Sigma Y^2 - (\Sigma Y)^2]}}.\end{aligned}$$

Since X and Y may be any scalar values, the items may be grouped into classes, and arbitrary values may be assigned to the classes, *e. g.*, $-2, -1, 0, +1, +2 \ldots$.

If in two or more groups the same arbitrary class values are used, the number of items in one group being n_1, in another n_2, etc., the correlation coefficient for each group may be obtained separately, and by a simple computation the correlation coefficient for the combined groups may be obtained:

$$r = \frac{(n_1+n_2+\ldots)\Sigma(\Sigma XY) - [(\Sigma X)(\Sigma Y)]}{\sqrt{\left\{(n_1+n_2+\ldots)\Sigma(\Sigma X^2) - \Sigma[(\Sigma X)^2]\right\}\left\{(n_1+n_2+\ldots)\Sigma(\Sigma Y_2) - \Sigma[(\Sigma Y)^2]\right\}}}.$$

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PROGRESS OF THE CENSUS OF AGRICULTURE

Of the various schedules used for collecting the information covered by a decennial census, the farm or agricultural schedule is the most elaborate. It provides spaces for more than four hundred different items, although of course only a small proportion of these items needs to be filled out for any one farm. The Director of the Thirteenth Census (1910) expressed the opinion that the number of inquiries on the schedule used at that census should be reduced by one-half or two-thirds. But probably no such reduction as that could have been made without lessening materially the value of this important branch of the census inquiry. The number of items, however, was reduced somewhat at the present census, there having been more than five hundred in 1910.

The schedule contains 8 questions regarding the farmer or farm operator; 7 regarding the form of tenure; 10 as to acreage, the acreage being classified with respect to the kind of land and the purpose for which it is used; 3 as to the value of the farm; 3 as to encumbrances on the farm; 4 as to expenditures for feed, labor, and fertilizer; 4 as to irrigation; and 5 as to drainage. Then follows an inventory of farm animals classified not only by kind but with respect to age also, so as to distinguish the growing colts and calves from mature horses and cattle. By far the greater part of the schedule, however, is taken up with the subject of farm products; and it is the variety and the number of these products which make the schedule so elaborate. It distinguishes 13 different hay and forage crops; 19 grain and feed crops; 11 varieties of vegetables; 16 fruit crops; 6 other field crops, including such important crops as cotton and tobacco; and outside of all these, the potato, sugar, and nut crops. Then there are the various dairy and meat products of the farm to be accounted for—the poultry, eggs, milk and butter and cheese, the pigs and calves slaughtered, etc.

As a rule three items are asked for under each crop, viz., the acreage harvested, the quantity harvested, and the quantity sold and to be sold. In former censuses the value of the crop was also called for; at this census that item of information has been omitted, but the values of the crops will be estimated on the basis of average prices obtained through the Department of Agriculture. Taking the place of the question as to value is that as to the quantity of the crop sold or to be sold, which in its general application is a new feature of the agricultural schedule introduced at this census. Another new subject of inquiry is that of farm facilities, questions being included regarding the number of tractors, automobiles and motor trucks, and whether the farm has a telephone, water piped into the house, and gas or electric light.

The number of farm schedules filled out at this census was approximately 6,500,000, the exact number of farms tabulated being 6,485,160.

For the first time in the history of the census the farm schedules have been tabulated by the use of the punch card and electrical tabulating machines. It is very evident, however, that no single punch card will provide for all the items on this elaborate schedule. It was found necessary, in fact, to use 17 different cards, each covering a different portion of the schedule. In the aggregate 142,696,217 cards were punched.

The card punching and tabulation have been completed and the publication of the results in the form of state bulletins has been begun. Bulletins have already appeared for 20 states and for Hawaii, Alaska, and the District of Columbia, and it is expected that the series will be completed by the first of September. The state bulletin contains 5 county tables covering the following subjects:

1. Farms and farm property
2. Live stock on farms and ranges
3. Domestic animals not on farms or ranges
4. Value of all crops, and acreage and production of principal crops
5. Mortgage debt and expenditures for labor, fertilizer and feed

Altogether this series of tables will present over 300 items of information for every county in the United States. On the subjects which these tables cover the Bureau does not contemplate publishing any further details for either states or counties, and, therefore, so far as these subjects are concerned—and they comprise the main part of the agricultural schedule—the census figures will all be "out" as soon as this series of bulletins is completed. But in the final volumes of the census reports the statistics will be reassembled and presented by subjects primarily instead of by states.

There are, however, a number of subjects which are not covered by these state bulletins and on which special bulletins or monographs will probably be issued later. These include pure bred live stock, farm facilities, coöperative marketing, forests and forest products, young animals raised on farms, domestic animals slaughtered on farms, nurseries, and green-houses.

Mention should be made also of the subjects of irrigation and drainage, which are closely related to agriculture though not strictly a part of the census of agriculture and not covered by the agricultural schedule. Previous censuses have covered the subject of irrigation, but this is the first time that the subject of drainage has been included. Two state bulletins have been issued on irrigation, and five on drainage.

J. A. Hill

STATURE AND WEIGHT OF CHILDREN

The Children's Bureau has recently issued basic tables of heights and weights for white children under six years of age, in advance of publication of a final bulletin. These tables show average heights and weights by age and sex from birth to 72 months, and average weights for children of different heights. They are based upon measurements of 167,024 white children. The records included in the tabulation were carefully selected from over 2,000,000 records received by the Bureau during the Children's Year Campaign. The basis of selection was fourfold: (1) the record card must have been signed by a physician; (2) no serious defects should have been noted; (3) the child must have been weighed and measured without clothing, as verified by the physician signing the card; (4) all essential items must have been answered. The children included in the tabulation were 76 per cent of native mothers, four per cent of British and Irish parentage, and the remainder of Scandinavian, Italian, and other racial stocks. They represent children from every part of the country. The averages for white children of native parentage agree very closely with these averages for all white children.

The full report, it is announced, will contain comparisons of rural and city children, of children of native parentage, children of Italian, German, and Scandinavian parentage, and a study of the correlation between the presence of certain abnormal conditions and stature and weight.

TABLE I

STATURE AND WEIGHT OF WHITE CHILDREN FROM BIRTH TO 6 YEARS OF AGE, SMOOTHED FIGURES

Age	Boys		Girls	
	Stature (inches)	Weight* (pounds)	Stature (inches)	Weight* (pounds)
Under 1 month	21.16	9.11	20.89	8.65
1 month, under 2	22.47	10.88	21.92	10.14
2 months, under 3	23.58	12.61	23.09	11.71
3 months, under 4	24.55	14.07	24.00	13.03
4 months, under 5	25.38	15.37	24.83	14.25
5 months, under 6	26.10	16.50	25.53	15.32
6 months, under 7	26.72	17.47	26.15	16.26
7 months, under 8	27.27	18.31	26.70	17.09
8 months, under 9	27.76	19.04	27.20	17.81
9 months, under 10	28.21	19.68	27.66	18.45
10 months, under 11	28.64	20.27	28.10	19.03
11 months, under 12	29.06	20.81	28.51	19.56
12 months, under 13	29.47	21.32	28.91	20.04
13 months, under 14	29.87	21.82	29.30	20.51
14 months, under 15	30.26	22.31	29.69	20.96
15 months, under 16	30.64	22.78	30.08	21.42
16 months, under 17	31.02	23.25	30.47	21.88
17 months, under 18	31.39	23.72	30.86	22.35
18 months, under 19	31.76	24.18	31.24	22.81
19 months, under 20	32.11	24.62	31.60	23.26
20 months, under 21	32.44	25.04	31.93	23.68
21 months, under 22	32.76	25.45	32.24	24.07
22 months, under 23	33.06	25.84	32.53	24.45
23 months, under 24	33.34	26.21	32.81	24.81
24 months, under 25	33.62	26.58	33.09	25.17
25 months, under 26	33.89	26.95	33.37	25.54
26 months, under 27	34.16	27.31	33.66	25.92
27 months, under 28	34.43	27.68	33.95	26.31
28 months, under 29	34.71	28.06	34.24	26.70
29 months, under 30	34.99	28.45	34.53	27.09
30 months, under 31	35.27	28.83	34.82	27.48
31 months, under 32	35.54	29.20	35.09	27.85
32 months, under 33	35.79	29.56	35.34	28.20
33 months, under 34	36.02	29.88	35.58	28.54
34 months, under 35	36.24	30.18	35.81	28.88
35 months, under 36	36.46	30.47	36.03	29.20
36 months, under 37	36.67	30.75	36.25	29.51
37 months, under 38	36.89	31.04	36.48	29.82
38 months, under 39	37.11	31.34	36.72	30.13
39 months, under 40	37.34	31.67	36.96	30.46
40 months, under 41	37.57	32.01	37.21	30.79
41 months, under 42	37.81	32.36	37.46	31.12
42 months, under 43	38.05	32.71	37.70	31.44
43 months, under 44	38.29	33.05	37.92	31.73
44 months, under 45	38.51	33.36	38.13	32.00
45 months, under 46	38.72	33.65	38.33	32.27
46 months, under 47	38.91	33.91	38.52	32.53
47 months, under 48	39.09	34.14	38.70	32.78
48 months, under 49	39.25	34.36	38.89	33.05
49 months, under 50	39.42	34.59	39.08	33.31
50 months, under 51	39.59	34.82	39.27	33.57
51 months, under 52	39.77	35.06	39.47	33.84
52 months, under 53	39.96	35.33	39.69	34.17
53 months, under 54	40.18	35.63	39.91	34.48
54 months, under 55	40.40	35.94	40.14	34.81
55 months, under 56	40.62	36.27	40.36	35.14
56 months, under 57	40.83	36.61	40.58	35.45
57 months, under 58	41.03	36.94	40.79	35.75
58 months, under 59	41.22	37.28	40.98	36.05
59 months, under 60	41.40	37.60	41.16	36.35
60 months, under 61	41.57	37.91	41.34	36.65
61 months, under 62	41.75	38.20	41.52	36.94
62 months, under 63	41.93	38.48	41.69	37.23
63 months, under 64	42.11	38.75	41.86	37.52
64 months, under 65	42.31	39.05	42.03	37.80
65 months, under 66	42.51	39.36	42.20	38.09
66 months, under 67	42.71	39.69	42.38	38.42
67 months, under 68	42.91	40.03	42.58	38.74
68 months, under 69	43.10	40.39	42.81	39.14
69 months, under 70	43.32	40.72	43.03	39.53
70 months, under 71	43.48	41.17	43.39	40.17
71 months, under 72	43.87	41.40	43.52	40.36

* Weights do not include clothing.

TABLE II

AVERAGE WEIGHT FOR HEIGHT, BY SEX; WHITE CHILDREN *

Stature (inches)	Boys		Girls	
	Number	Average weight† (pounds)	Number	Average weight† (pounds)
20	206	8.19	310	8.13
21	486	9.47	654	9.28
22	905	10.55	1,121	10.53
23	1,352	11.94	1,635	11.88
24	1,994	13.47	2,318	13.33
25	2,496	15.01	2,866	14.82
26	3,068	16.60	3,179	16.35
27	3,525	18.03	3,561	17.66
28	3,775	19.39	3,471	19.01
29	3,605	20.68	3,425	20.20
30	3,671	21.99	3,563	21.30
31	3,834	23.24	3,750	22.60
32	3,803	24.56	3,805	23.81
33	4,075	25.80	3,951	25.06
34	4,533	27.12	4,405	26.41
35	4,698	28.41	4,596	27.75
36	5,159	29.72	4,919	29.13
37	5,074	31.09	4,974	30.37
38	5,265	32.44	4,973	31.71
39	5,133	33.84	4,823	32.99
40	4,769	35.28	4,505	34.49
41	4,181	36.83	3,894	36.02
42	3,393	38.38	3,002	37.49
43	2,312	40.13	2,020	39.23
44	1,598	41.73	1,341	40.94
45	857	43.50	703	42.79
46	423	45.41	349	45.00
47	193	47.29	143	46.85

* Averages are not shown for statures under 20 inches nor for statures 48 inches or over, since the numbers of children of these statures were too small to afford satisfactory averages.
† Weights do not include clothing.

R. M. WOODBURY

STATISTICAL ACTIVITIES IN CENTRAL EUROPE

The governments of Central Europe are not falling materially behind in their statistical publications. Regardless of the frightful economic consequences of the war, most of the countries, states, and cities have continued to issue periodical reports of the utmost practical value in the furtherance of plans of reorganization and rehabilitation. Particularly is this true of Austria, which, in spite of appalling economic disadvantages, has not neglected the first duty of the state to keep itself and the public properly informed regarding the essentials of community life as represented in the current statistics of collective phenomena. During 1918 there was published the *Yearbook of Austrian Cities*, by the Statistical Central Committee, containing a wealth of information in the most convenient form, which, if available for American cities, would unquestionably materially advance the cause of good government. During the same year, and regardless of the disorganization common to all Central European countries during the latter years of the war, there was issued the thirty-fifth volume of the *Austrian Statistical Yearbook*, by the Statistical Central Commission, for the period 1916–17, including the latest available information on all the essentials of the social and economic conditions of what formerly was the Austrian Empire. The publications of the Austrian census of 1910 were also issued regardless of the

war, and it is practically a certainty that for many years to come these reports for the separate constituent parts of the late Austrian monarchy will constitute a most useful and trustworthy source of information.

But the most extraordinary contribution to statistical literature is the *Preliminary Report of the Census of the Austrian Republic*, as of January 31, 1920, communicated to the government by the president of the Statistical Central Commission under date of May, 1920, comprehending a brief analysis of all the essential facts of population, housing, war prisoners, etc., with comparative data for certain foreign countries. The rapid manipulation of the statistical material reflects credit upon the statistical authorities who are confronted by many difficulties of an extraordinary nature. The total population of what is now the Austrian Republic, which had increased from 5,650,000 in 1900 to 6,294,000 in 1910, had decreased by 1920 to 6,067,000, or, in round figures, 227,000, or 3.61 per cent. For the large cities the decrease in population during the decade ending with 1920 was 9.33 per cent for Vienna, and 9.68 per cent for Klagenfurt. Other cities show an increase, but of minor significance contrasted with the very material decrease in the case of the city of Vienna.

The male population of the Austrian Republic during the decade ending with 1920 decreased 6.61 per cent, while the female population decreased 0.68 per cent; but in the case of the city of Vienna alone the decrease in male population was 12.52 per cent, against 6.39 per cent for the female population.

The accentuated housing condition is illustrated by an increase of only 1.15 per cent in housing accommodation during the period ending with 1920, against an increase of 7.23 per cent during the decade ending with 1910.

The questionnaire required information regarding war prisoners and persons missing or unaccounted for as a result of the war. The number of Austrian war prisoners still absent from home and held in some foreign country was 19,378, or 3.19 per 1,000 of population. The number missing or unaccounted for prisoners as a result of the war was 22,351, or 3.68 per 1,000. These reports, however, are considered as falling below the truth.

Reference is made in the report to the Census of Bavaria as of October 8, 1919, indicating an actual decrease in the male population of 20,936. The census for Prussia as of October 8, 1919, indicated an actual increase in population of 727,000, while in Saxony there was an ascertained decrease during the preceding decade of 138,000, or 2.9 per cent. For the city of Berlin the census of October 8, 1919, returned a population of 1,902,000, against 2,071,000 as of December 1, 1910. During practically the same period, however, the population of Munich increased at the rate of 4.3 per cent. These figures would clearly indicate that Austria suffered more in population losses than Germany or any of its larger constituent parts.

In addition to the foregoing statistical publications the Austrian Statistical Central Commission has commenced the issuing of a new series of statistical yearbooks, of which the first is for the year 1920, issued under date of October over the signature of the president, Dr. Victor Mataja. This extremely valuable publication contains, aside from geographical data and demographic information, statistics concerning mortality, agriculture, forestry, animal husbandry, mining, industries, labor organizations, transportation, public and private credit, prices, labor conditions, law and justice, education, and public finance. In many cases the statistical information ends, however, with the year 1917–18, but there are important exceptions in which the data are brought down to the end of 1919. The demography ends with 1918, clearly indicating the extraordinary rise in mortality as a result of war conditions. The number of deaths from all causes in the Austrian Republic, returned as 119,462 for the year 1914, had increased to 166,378 in the year 1918. Even more suggestive

are the statistics of birth, the number of live births having diminished in the Austrian Republic from 153,542 in 1913 to 87,594 in 1918. Of interest in this connection is a statement regarding prices, or the cost of living, best indicated perhaps by the actual price in heller of a kilogram (2.2 lbs.) of bread, the cost of which in the city of Vienna increased from 32 heller in 1914 to 476 by June, 1920. Even more startling is the increase in the price of a kilogram of flour, from 44 heller in 1914 to 1,120 in 1920. Yet these increases are insignificant when contrasted with the enormous rise in the price of lard, from 190 heller a kilogram in 1914 to 11,000 in 1920; of potatoes, from 24 heller in 1914 to 860 in 1920; of sugar, from 84 heller in 1914 to 4,200 in 1920; of roast beef, from 195 in 1914 to 9,800 in 1920; and finally, of pork, from 200 heller in 1914 to 15,000 heller per kilogram in 1920.

The number of unemployed persons, or persons seeking work, increased from 244,000 in 1918 to 648,000 in 1919. The number of persons receiving out-of-work allowances increased from 46,000 on December 1, 1918 to 87,000 on November 22, 1919, having attained to a maximum of 186,000 by May 1. In Vienna alone the number of persons out of work increased from 24,000 on December 1, 1918, to 73,000 on November 17, 1919.

The sick-fund statistics are available only for the year 1915, when 1,049,000 persons were insured, among whom there occurred 349,000 cases of sickness, involving a labor loss of 6,800,000 days and a mortality of 10,867. These sick funds had an income of 33,000,000 kronen, of which 21,000,000 was paid by the workmen, 9,399,000 by the employer, and the remainder from interest or other sources. The amount disbursed was 28,400,000 kronen, of which 11,000,000 was paid out in cash benefits, 6,000,000 for medical attention and control, 2,500,000 kronen for drugs, etc., 2,400,000 for institutional care, 924,000 kronen for funerals, and 4,200,000 for administration.

The foregoing is a brief resumé of what in the field of statistics must be considered an extraordinary achievement in view of the limitations of clerical help, the political disorganization, the want of funds, the cost of printing, etc., etc. Nothing, however, could more clearly emphasize the unhappy economic condition of the Austrian Republic while suggestive of the world's sympathy as the victim of imperial misrule and the sinister designs of the Prussian monarchy.

F. L. Hoffman

MEASURING THE BUYING POWER OF THE COMMUNITY

A Business Research meeting of the American Statistical Association was held at a dinner on February 18 at the Machinery Club in New York City. Ernest S. Bradford, who presided at the meeting, emphasized the general interest of this topic, and stated it to be the problem of business research to determine the amount of goods wanted at a particular place, as of a specified time, and how to get them there.

Dr. H. Parker Willis, discussing "Business Indices as a Factor in Measuring Buying Power," pointed out the importance of scientific method in business and the essential character of analysis of general business conditions. He explained briefly the indices which are being developed in the Federal Reserve Board at the present time, and which have a bearing on measurment of buying power, as follows:

(1) General business activity in each Reserve District as reflected by certain statistics of bank transactions. Debits to individual accounts are telegraphed weekly to the Federal Reserve Board from 161 centers. The speaker gave the opinion that these figures are superior to clearing house data.

(2) Retail trade activity as reflected from the voluntary monthly reports to the

Federal Reserve Board from about 200 department stores situated in various parts of the country.

(3) Variation in production and consumption as reflected by the physical volume of trade series, developed from reports of production, movements, or stocks on hand of certain staples in selected leading industries.

(4) A collection index, intended to show the rapidity of the turnover of funds. (For method, see article by W. H. Steiner in the December 1920 issue of the AMERICAN STATISTICAL QUARTERLY).

(5) Series on the movements in foreign trade, presenting separate indices for exports and imports.

Mr. Chester I. Barnard, director of research in the Ratemaking Department of the Bell System, discussed the methods of measuring future demand for telephones. Forecasts of the demand for residence telephones are based upon the relation between the demand for service and the standard of living. The latter is reflected in general by the mean rent of resident property. The racial and occupational composition of the population, and any other factors which contribute to fundamental changes in industry or the general make-up of the community, must be considered. This involves an estimate of the character of the population and the general standard of living, by small homogeneous areas, for a period of twenty years in the future. The object is to estimate (1) the number of telephones required in the future, for a period even twenty years hence, and (2) how this demand will be distributed by small local areas. This enables the company to locate properly the central office for a district and the main leads. Research work of this character has been going on in the American Telephone and Telegraph Company for years.

Mr. J. C. Luitweiler, of the American International Corporation, outlined the methods used by them in measuring the buying power for coffee. The problem was approached by basing future estimates upon past actual consumption and making allowance for the present curtailed buying power. The study was divided into three parts: (1) The general picture of coffee consumption in the United States and in foreign countries for a period of the past ten or twelve years; (2) estimate of the present buying power in comparison with past buying power; (3) the extent to which the present limited buying power will be devoted to the purchase of coffee.

Mr. W. H. Berridge, representing the Harvard Committee on Economic Research, discussed the relation of the condition of employment to the buying power of the labor group. A time series of unemployment was related to various other series of data and was regarded as indicative of changes in buying power.

Briefer discussion of the general topic was presented by Andrew H. Melville, director of research for the Nast Publications, who considered the subject from the standpoint of the magazine publishers, pointing out that their interest in measuring buying power centers around the selling of advertising space. Alice C. Boughton, director of research for the J. Walter Thompson Company, from the viewpoint of the advertising group, stated their primary interest to be securing purchasers for a particular product. Joseph E. Pogue, consulting engineer, pointed out the importance of the use of indices in comparing actual with potential performance, sales with sales possibilities; and in forecasting demand. The former is useful in checking past operations, the latter in planning ahead. T. K. Stevenson, sales statistician for the Western Electric Company, emphasized the importance of determining the portion of buying power which one could reasonably claim for himself, as distinguished from the measurement of total buying power. This will vary for different territories, depending upon competition and comparative contiguity to distributing warehouses.

In the general discussion from the floor emphasis was placed upon the importance of the time series in the control of business. In this manner the general trend of a business can be shown and especially the condition of the industry prevailing in time of depression. Few classes of industry fall more than 70 per cent below the normal. In a summary of the results of the discussion it was pointed out that faith in research is quite as important as imagination and technical equipment. Business must be willing to spend money for research whether the results be negative or positive. Statistical and economic prospecting is coming to be a very necessary function of a business organization.

R. E. CHADDOCK

FIRST FEDERAL CONFERENCE ON BUSINESS CONDITIONS REPORTING

On February 21–22 there was held under the auspices of the Division of Analysis and Research of the Federal Reserve Board, the first Federal Conference on Business Conditions Reporting, at the Hotel Washington, Washington, D. C.

The work of the conference was divided into two parts. On the first day representatives of each of the departments of the Federal Reserve Agents were present and various problems relating to the scope of the monthly review on business conditions which each Federal Reserve Bank prepares were discussed, as well as the experience of the banks with different methods of obtaining information presented in these reviews. The interchange of views which occurred resulted in making available to each bank the experiences of every other bank in connection with specific points which confront it, as well as indicating new lines of work that are being pursued in other banks and the results and difficulties in connection with such new lines of endeavor. Agreement was reached on some leading questions in connection with the preparation of the reviews, and a committee was appointed to carry on the work commenced at the conference.

On the second day there were also in attendance a number of leading business statisticians and men in academic circles interested in this type of work. It seemed desirable to obtain the opinions of "consumers" of data of this kind as to just what type of material was desirable and in what manner such material should be presented. To this end there were papers read, among others, by Professor M. T. Copeland, of Harvard University, on "The Importance of Securing Reliable Statistics on Business Conditions"; by Mr. Robert M. Falkenau, foreign trade representative of the Irving National Bank of New York, on "The Value of Work on Business Conditions to the Economic Advisers of Banks"; by Mr. F. Leslie Hayford, statistician of the General Motors Corporation, on the "Following of Business Conditions by the Corporation"; and by Mr. Carl Snyder, manager of the Statistics Department of the Federal Reserve Bank of New York, on "Interpretation of Reports on Business Conditions." General discussion was led by Professor Edmund E. Day of Harvard University. At the afternoon session it was planned to consider the actual methods employed in the preparation of certain statistical data relative to business conditions, in particular indicating the methods employed and the problems experienced by some of the Federal Reserve Banks. By thus indicating the work of the Federal Reserve System a better understanding of its efforts would be obtained, and even more complete coöperation than heretofore would be secured from the business community. The Honorable W. P. G. Harding, governor of the Federal Reserve Board, first addressed the conference on the subject of "Business and Banking," emphasizing the value of data on business conditions to the banking and the business community. The statistics on

business activity assembled by the Division of Analysis and Research were then described by Dr. H. Parker Willis, director of the Division, and the banking statistics of the Federal Reserve Board by Mr. M. L. Jacobson, statistician. Mr. R. A. Cheney, secretary of the Association of Knit Goods Manufacturers, described the statistical work which the Association undertook some time ago and spoke of the interest which the industry took in these figures of production, shipments, etc. Mr. Henry Breck, assistant Federal Reserve Agent of San Francisco, then described the methods employed by his bank in preparing its monthly reports on wholesale and retail trade, and Mr. G. B. Roberts of the Federal Reserve Bank of New York described the preparation of the reports on financial conditions issued by his bank. Mr. W. H. Steiner, assistant director of the Division of Analysis and Research, reviewed the work of the Division and the special studies in Banking and Credit.

Following the presentation of the formal papers at both the morning and afternoon sessions on the second day, there was general discussion which developed many points of interest. It was generally felt that the Federal Reserve Board was in an excellent position to obtain information relative to business activity, and that the work which it had thus far undertaken should be actively prosecuted and expanded in the future.

W. H. Steiner

MISCELLANEOUS NOTES

Index of Retail Food Prices. The Bureau of Labor Statistics of the United States Department of Labor announces that hereafter its index to retail food prices will be based upon 43 articles of food and weighted according to the quantity of each article consumed by the average workman's family, as shown by the 9,000 budgets secured in 1918, instead of being computed as heretofore on the average family market basket, based upon a study made in 1901.

Since January, 1919, the Bureau has been securing retail prices upon the 43 articles of food to which its new weighting system is applied. These figures are secured on the 15th of each month from retail dealers in 51 of the most important cities. Until January, 1921, however, only 22 of the articles could be used in the cost of living index because there was no available information as to the average family consumption of the remaining articles. The family budget investigation of 1901–02 did not give the food expenditures in sufficient detail to justify a weighting upon more than the 22 articles (therefore the other articles had to be combined and used as "all others"). In the family budget investigation of 1918 a more detailed distribution of food expenditures was sought, and this gives an annual consumption per family, which can be used as food weights, on a much broader basis. Beginning with the March issue of the *Monthly Labor Review*, in which the January prices will be published, the figures will be based on the 43 articles of food. It must be understood that these new figures merely give a more detailed distribution of the total food costs. It is interesting to note that in the adjustment of the 43 articles of food to those already used by the Bureau for the 22 articles, the variation in the index number was but four-tenths of one per cent.

Index of Wages. The Bureau of Labor Statistics of the United States Department of Labor has made public a general index of wages per hour from 1840 to 1920 based upon hourly wages in 1913 as 100. It is fully realized that the material for computing such an index is disconnected and far from complete. For many years there has been hesitation as to the propriety of announcing an index of this character, but the importance of having a general wage index that might be used for comparison

with similar index number changes in cost of living and wholesale prices has determined the Bureau to go over the mass of material and prepare such an index. Although not claiming for this the same degree of accuracy that attaches to the work of the Bureau in investigations where all the material is gathered and compiled within the Bureau, yet it is believed that the table is based upon data essentially correct. At all events, the very best available material has been used and the table is the best that can be constructed upon this material. The tabulation showing index numbers of wages per hour, 1840 to 1920, is as follows:

INDEX NUMBERS OF WAGES PER HOUR, 1840 TO 1920
(Currency basis during Civil War period)
(1913 100)

Year	Index number	Year	Index number
1840	33	1910	93
1850	35	1911	95
1860	39	1912	97
1865	58	1913	100
1870	67	1914	102
1875	67	1915	103
1880	60	1916	111
1885	64	1917	128
1890	69	1918	162
1895	68	1919	184*
1900	73	1920	234†
1905	82		

* This index number applies to the spring of the year. Wage rates advanced during the year.
† This index number applies to the summer of 1920, and probably represents the wage peak of the year.

International Labor Review. The first number of the *International Labor Review* has been received in this country. It is edited by the International Labor Office under the League of Nations. In the first issue, January, 1921, there are a number of special articles and, in addition, six sections: Industrial Relations; Production, Prices and Cost of Living; Unemployment; Protection of Women and Children; Coöperation; and Agriculture. The special articles include accounts of the International Labor Organization by Albert Thomas, its director, and of the International Trade Union movement and its relation to the Labor Office, by J. Oudegeest, secretary of the International Federation of Trade Unions. Sidney Webb contributes an article on the "Process of Amalgamation in British Trade Unionism," and John R. Commons discusses industrial government. This issue contains 150 pages and is devoted to many different aspects of problems of labor in various countries.

The Advisory Census Committee. The Advisory Committee to the Director of the Census, appointed in 1919 by the American Statistical Association and the American Economic Association at the request of the Secretary of Commerce, has been asked to assume additional duties. At the request of the Chairman of the Senate Committee on the Census, it has taken under consideration the methods of apportioning representatives. The committee met in Washington on March 11 and 12 to examine the mathematical and statistical aspects of the problem. The meetings were attended also by Professors A. A. Young, E. V. Huntington, Irving Fisher, F. W. Owens, and Dr. J. A. Hill. A report to the Senate Committee is now in preparation. This same committee, with the addition of Professor A. A. Young, will serve also as an Advisory Committee on Statistics to the Department of Commerce, in compliance with a request from Secretary of Commerce Hoover. Two annual re-

ports of the Advisory Committee have been published in the QUARTERLY, Vol. XVLI, pp. 76–107 and 465–83.

Occupational Classification of Railroad Positions. The United States Railroad Labor Board recently prepared a comprehensive functional occupational classification of railroad positions as a basis upon which monthly reports are to be made by railroads to the Interstate Commerce Commission and to the Labor Board. With the establishment of this classification and the institution of the reporting forms and rules which the Board has developed in addition to the classification proper, railway wage statistics will be put upon a sound statistical basis and the data reported will be made of service for administrative and public purposes. The Interstate Commerce Commission has formally approved the forms and rules and has issued a wage order making them effective for reports on July 1, 1921. It has also approved the classification and has made it the basis for assigning positions in the reports.

A Clothing Survey. The clothing survey made in coöperation with the National Association of Retail Clothiers by the Bureau of Business Research of the Northwestern University School of Commerce, under the direction of Dr. Horace Secrist, has been completed. The report of this survey is to be published in six volumes, and will cover such topics as Sales and Sales' Ratios in Retail Clothing Stores; Expenses and Expense Ratios, including rent, wages, bushelling, and general and total expenses; Advertising Methods and Expenses; Purchase Discounts, Inventories, Average Stock and Stock Turnover; and also a study of clothing stores as operating units.

Public Health Committee. The Committee of the American Public Health Association has for some time been collecting information regarding the public health procedures in American cities. The field work is now practically completed and surveys of eighty-three cities are being compiled. The Committee will issue its report within the calendar year.

A special dinner-meeting of the American Statistical Association was held in the dining rooms of the Metropolitan Life Insurance Company, New York City, on Friday evening, April 15. Sir Arthur Newsholme, K. C. B., an honorary member of the Association and lecturer on Public Health Administration at the Johns Hopkins University, formerly Principal Medical Officer of the Local Government Board of London, was a guest of the Association and spoke on the topic, "The Better Use of Vital Statistics in Public Health Administration." In his address Dr. Newsholme spoke with keen insight of the public health administration in the United States and showed a sympathetic understanding of our successes, difficulties, and shortcomings. At the conclusion of the address, one after another of the members of the Association arose to pay tribute to Dr. Newsholme and the services he has rendered the cause of public health. Dr. Newsholme has been in this country for two years, lecturing at the School of Hygiene and Public Health of the Johns Hopkins University. He has also visited various states and has been a great constructive force in stirring up public health activities. He sailed for England on April 16.

A special meeting and dinner of the Washington members of the American Statistical Association was held in Washington, D. C., on April 8. The program consisted of two papers: "National Income," by Wesley C. Mitchell, director of the National Bureau of Economic Research, New York; and "A Proper Distribution of Credit between Consumption and Investment," by Franklin T. Miller, Washington, D. C.

Dr. R. B. Robbins, assistant professor of Mathematics and Insurance at the University of Michigan, is on leave of absence during 1921 and is now serving as assistant actuary in the office of the Superintendent of Insurance of the State of New York at Albany.

Mr. William M. Steuart has been appointed director of the Census to succeed Mr. Samuel L. Rogers.

James W. Glover, professor of Mathematics and Insurance at the University of Michigan, has been made a member of the Committee on Mathematical Analysis of Statistics, originally appointed by the National Research Council.

Mr. F. Leslie Heyford, formerly in the Wilmington office of the Du Pont de Nemours Company, has joined the staff of the General Motors Corporation in New York City.

The editors announce that commencing with the June issue of the QUARTERLY PUBLICATIONS the Association will furnish without charge fifty reprints to contributors of articles.

COMMITTEE APPOINTMENTS

Committee on Fellows:

John Koren appointed for five years, to take the place of Frederick L. Hoffman, whose term expired 1920.

The full committee is as follows:

Adna F. Weber,	term expires		1921
Edward L. Thorndike,	"	"	1922
Carroll W. Doten, Chairman,	"	"	1923
Leonard P. Ayres,	"	"	1924
John Koren,	"	"	1925

Committee on Institutional Statistics:

Edith Abbott
Kate H. Claghorn
Edith M. Furbush
Joseph A. Hill
Robert M. Woodbury
Horatio M. Pollock, Chairman

Joint Committee on Statistics of Marriage and the Family:

Edwin W. Kopf, representing the Statistical Association.

Committee on Business Research:

Ernest S. Bradford, formerly director of Business Research, The McGraw Hill Co., organizing Chairman of the Committee
Chester I. Barnard, American Tel. & Tel. Company, representing the Public Utilities group
C. S. Duncan, Southern Wholesale Grocers' Association, representing the Merchandising group
L. K. Frank, New School for Social Research, representing the Economic Research group
F. Leslie Hayford, General Motors Corporation, representing the Manufacturers' group
C. A. Hoppock, H. K. McCann Company, representing the Advertising Agencies
A. H. Melville, The Nast Publications, representing Publishers

E. H. Naylor, Writing Paper Manufacturers' Association, representing Trade Associations

Richard Webster, Geo. Batten Company, representing Advertising Agencies

H. Parker Willis, Division of Analysis and Research, Federal Reserve Board, representing Banking and Credit Institutions

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NEW MEMBERS ELECTED SINCE JANUARY 1

Corporate Members

Metropolitan Life Insurance Company, 1 Madison Ave., New York

Washburn-Crosby Company, Minneapolis, Minn.

Regular Members

Alford, Clarence L., 1108 Independent Life Bldg., Nashville, Tenn. Statistical work with Mr. Abb Landis

Alldredge, E. P., secretary, Survey, Statistics & Information, Southern Baptist Convention, Nashville, Tenn.

Anstie, Miss Lillian F., S. K. F. Industries, Inc., 165 Broadway, New York. Preparation of financial statistics

Anderson, Leo Dewey, Bureau of Personnel Research, Carnegie Institute of Technology, Pittsburgh, Pa.

Bernstein, Alton F., Public Service Commission, 48 Lafayette St., New York

Bivins, Percy A., The Standard Textile Products Co., 320 Broadway, New York

Blood, John Balch, 804 Interstate Commerce Bldg., Washington, D. C. Valuation Analyst on Railroads, Interstate Commerce Commission

Brissenden, Paul F., New York University, Washington Square, New York. Assistant Professor of Economics

Brown, Lathrop, the St. James, New York

Brown, Theodore H., Dept. of Statistics, Brown University, Providence, R. I.

Bullis, H. A., Chief Statistician, Washburn-Crosby Co., Minneapolis, Minn.

Burgess, Mrs. May Ayres, secretary, Education Dept., Russell Sage Foundation, 130 E. 22d St., New York

Camp, Burton H., Dept. of Mathematics, Wesleyan University, Middletown, Conn.

Cassidy, Henry L., American Statistical Service Co., Holyoke, Mass.

Cathles, Laurence M., vice president and actuary, Southland Life Ins. Co., Dallas, Texas

Cole, Fred J., public accountant, 1237–38 City National Bank, Omaha, Neb.

Cowen, A. Y., Miller & Co., 120 Broadway, New York

Davis, Robert M., statistical editor, *Electrical World*, New York

Foster, William T., director, Francis D. Pollak Foundation for Economic Research, 109 Sargent St., Newton 58, Mass.

Freyd, Max, research fellow, Bureau of Personnel Research, Carnegie Institute of Technology, Pittsburgh, Pa.

Gantt, Miss Margaret E., Statistical Bureau, Metropolitan Life Ins. Co., 1 Madison Ave., New York

Hart, William A., Sales & Market Analysis, Frank Seaman Advertising Agency, 470 Fourth Ave., New York

Hartke, Geo. W., general accounting analysis, 400 Benedum-Trees Bldg., Pittsburgh, Pa.

Horney, Chas. J., 37 E. 28th St., New York. Business statistics

Hourwich, Iskander, Engineering Division, U. S. Army Air Service, McCook Field, Dayton, Ohio

Hulst, Miss Myra M., Health Service Dept., American Red Cross, Washington, D. C.

Hyde, Arthur J. W., secretary-treasurer, L. A. Thompson Scenic Railway Co., 220 W. 42nd St., New York

Jones, Herschel H., director, New York City Office, Division of Foods & Markets, 90 West Broadway, New York

Kattenhorn, Arthur W., Statistical Dept., American Tel. & Tel. Co., 195 Broadway, New York

Little, John Dutton, editor, United States Bulletin Service, 10 Jackson Place, Washington, D. C.

Magnusson, Leifur, Bank of Commerce, 31 Nassau St., New York

Michell, Humfrey, Dept. of Economics, McMaster University, Toronto, Canada. Member Royal Statistical Society

Mitscher, Harry A., Prudential Ins. Co., Newark, N. J.

Neubauer, Charles M., Byrnes & Baker, C. P. A., 52 Wall St., New York

Outhwaite, Leonard, Bureau of Industrial Research, 289 Fourth Ave., New York. Also instructor in Personnel Administration, Columbia University

Plimpton, R. E., chief, Service Dept., The Engineering Magazine Co., 120 W. 32nd St., New York

Raines, Lester C., 102 State Hall, University of Pittsburgh, Pittsburgh, Pa.

Read, Edmund D., Statistical Dept., American Tel. & Tel. Co., 195 Broadway, New York

Rich, Gilbert J., Dept. of Psychology, University of Pittsburgh, Pittsburgh, Pa.

Riggleman, John R., Instructor in Business Statistics, Harvard Graduate School of Business Administration, Cambridge, Mass.

Roa, Emeterio, 209 S. Thayer St., Ann Arbor, Mich. Student at University of Michigan

Rogers, Herbert W., instructor in Psychology, Psychological Laboratory, Yale University, New Haven, Conn.

Rose, William M., Market Investigation and Reports, 60 High St., Boston, Mass.

Ryan, Andrew H., director, Industrial Hygiene, Scovill Mfg. Co., Waterbury, Conn. Lecturer, Industrial Physiology, Yale University

Schuckman, F. E., Homestead Valve Mfg. Co., P. O. Box 1754, Pittsburgh, Pa.

Smead, E. L., chief, Division of Reports & Statistics, Federal Reserve Board, Washington, D. C.

Strong, Benjamin, governor, Federal Reserve Bank, 15 Nassau St., New York

Towns, Charles H., Low, Dixon & Co., 37 Wall St., New York. Investment banking

Tucker, E. H., 832 Van Nuys Bldg., Los Angeles, Calif. Economic statistician, with First Nat'l Bank, Los Angeles Trust & Savings Bank, and First Securities Company of Los Angeles, Calif.

Uhrbrock, Richard S., teaching fellow in Psychology, Carnegie Institute of Technology, Pittsburgh, Pa.

Working, Holbrook, University Farm, University of Minnesota, Minneapolis, Minn. Research in Agricultural Economics

REVIEWS

The Fall of the Birth Rate. A paper read before the Cambridge University Eugenics Society, 20 May, 1920. By G. Udny Yule. Cambridge: The University Press. 1920. 43 pp.

The lecture which is published in pamphlet form was planned, as the prefatory note explains, not as an original contribution but rather to offer to a non-technical audience "an exposition of matters already known and views previously expressed." In the first and more substantial part of the lecture Mr. Yule examines, in this spirit, the historical trend of British birth statistics since 1851. His discussion of the allowances which should be made, in the study of birth rates, for changes of age and of marital condition in the population serves well to exemplify two useful and accepted statistical methods—the standardized birth rate devised by Drs. Newsholme and Stevenson, and his own adaptation of Professor Tait's formula relating fertility with age. He reviews the interesting results of some recent attempts by English investigators to determine the different rates at which fertility has declined in different social classes. Proceeding thus, he arrives anew at familiar and generally approved conclusions: that a pronounced decline of the birth rate has been in progress in England since 1876; that it has been more marked in the upper and professional classes; and that it represents in the main a real decrease in the fertility of married women.

But at this point Mr. Yule breaks with prevailing opinion; for in interpreting the decline of fertility he concedes only a minor influence to the deliberate and volitional control of fertility by contraceptive practices. His argument in support of this position is superficial and unsatisfactory. It appears to rest largely on a very dubious inference from precarious evidence laid before the National Birth Rate Commission, and on a more than dubious interpretation of a reference in the report of the Commission to some of Mr. Sidney Webb's data on family limitation, first published in the London *Times* in 1906, and since then embodied in Fabian Tract No. 131 on *The Decline in the Birth Rate.* Mr. Yule seems not to have consulted the Webb study in the original, although he has ventured to cite it in support of a view at variance with the conclusions of both Mr. Webb and the Birth Rate Commission. He has virtually ignored the important foreign literature of his subject; and in many incidental ways he reveals that he is on unfamiliar ground, as, for instance, when he implies that voluntary control of births could not have had a significant influence on the French birth rate before 1875.

As affirmative argument in favor of his contention Mr. Yule adduces two conjectures to the effect that the fertility of a nation is subject to natural fluctuations. One is Dr. Brownlee's hypothesis of the variation of germinal vitality. The other is Mr. Yule's own belief in an undefined but significant relation between fertility and the economic cycle. What this relation is he frankly admits himself unable to state, though it seems to him a largely unconscious psychological response to economic conditions. The evidence of some of his earlier statistical studies suggests to him a correlation between national fertility and the general level of prices. But the correlation, as he recognizes, has latterly broken down; and one ventures to remark that even had it not broken down it would have been susceptible of many different interpretations.

No reflecting person would contest Mr. Yule's assertion that the fact that contra-

ceptive measures are used is not a satisfactory explanation of the motives for using them. Moreover, whatever be the reasons for their use, it is quite possible that we have been led to exaggerate the effect of such measures in lowering the birth rate. It is much to be desired that some highly qualified statistician should searchingly re-examine the record, and should challenge the prevalent interpretation if it appears to be mistaken. But if such a challenge is to be taken seriously it must be based on stronger evidence and sounder argument than this pamphlet offers us.

JAMES A. FIELD

The University of Chicago

Argentine International Trade Under Inconvertible Paper Money—1800–1900, by John H. Williams. Cambridge: Harvard University Press. 1920. xiv, 282 pp.

The author's object in writing this study of the international trade of Argentina has been to examine the inter-relations of three factors—paper money, borrowings, and trade—and thus to make inductive examination of the theory of international trade under a régime of depreciated paper. After an introductory analysis of the purpose of his study, he gives a long review of the monetary and commercial history of Argentina. This is not completed until the close of Chapter X. Thereupon he recurs to the theoretic side of the investigation and considers the effect of inconvertible paper on international trade balances. Following a chapter on that phase of the matter is a study of Argentine foreign trade which is continued up to the end of the book. Chapter XVI, entitled "Conclusion," restates a few of the main points and sets them forth in their theoretical relations. A detailed bibliography and a thorough index complete the volume.

It would probably have been better if Mr. Williams had simply presented his study as a history of Argentine currency and foreign trade during the years he has studied. The form in which the undertaking is stated at the outset and the suggestions thrown out from time to time in the course of the work give the reader the impression that the real or chief object of the writer is to afford a critical exposition of theories of foreign exchange and prices as "inductively" examined. The student of foreign exchange is thus impressed by the scantiness of the material supplied under these heads and is inclined, therefore, to adjudge the volume a disappointment when he reaches the end and finds the author still fencing with the theoretical issues involved and still disinclined to furnish the concrete conclusions promised at the outset. Yet to the reader who is willing to strip off the husks and take the study for its content the book has both merit as a piece of economic inquiry and value as an addition to commercial annals.

There is much detailed and careful review of Argentine history in this monograph, but it is scarcely practicable to present even the basic elements of it within brief space. The early monetary and commercial experience of Argentina is carefully sketched and then the author turns to a survey of public loans and the balance of "international payments." He next presents a history of the national banking system of Argentina and explains the origin of the speculative movement of the years 1885–1890. Turning then to the Baring Panic and the international financial troubles of the early nineties, the study passes in review the various factors which led to paper money depreciation. Later trade developments, the reformation of monetary conditions, and the development of individual exports are considered with equal care. It is a painstaking outline of the economic history of a country whose currency and price experience have been most interesting and whose lessons have been too little studied by English and American economists.

In so far as Mr. Williams reaches "theoretical" conclusions, they are of a nature which might about as well have been taken for granted at the outset. An example of this sort is seen in Chapter X on the relation between the value of paper money and the balance of international payments. Here he finds that the balance of international payments "did in fact exercise a powerful influence upon the value of Argentine inconvertible paper money." How any one could question this statement it would be difficult to see. This, as current experience in European exchanges clearly shows, is merely a special case of the law of demand and supply and the method of its operation, as has been repeatedly pointed out in current discussion of the exchange question. Another example of a similar, although not precisely parallel, sort is found in Mr. Williams' statement that Argentine experience in the export trade does not suffice to verify current theories with regard to the effect of a rising gold premium upon exports. Such a premium Mr. Williams believes has been currently regarded as furnishing a stimulus to exports. He finds that "such a stimulus was undoubtedly present," but owing to the operation of other factors the gold premium did not produce any marked increase in export trade; in other words, it was not possible to institute any such close comparison between the different classes of figures as had been desired.

Taking the book as a whole, it is concluded that data which have been surveyed indicate "a pretty general correspondence with theoretical reasoning." Especially clear-cut, the author thinks, is the relation between the balance of international payments and the value of inconvertible paper money. General conclusions with reference to the effect of the paper money experience upon the people of Argentina are expressly disclaimed as being "extremely difficult to verify." Expressions like these tend to stimulate in the reader's mind the thought that the book was originally projected upon too ambitious a scale and does not fulfil its promise, or at all events, fulfils it only in a negative way. This, as already stated, does not in the least detract from its merits as a careful study of economic conditions affecting the exchange and foreign trade of Argentina over the period to which the volume relates.

H. Parker Willis

Agricultural Prices, by Henry A. Wallace. Des Moines, Iowa: Wallace Publishing Company. 1921. 224 pp.

This book proposes a new method of determining the prices of agricultural products —a plan whereby the statistical economist and the production engineer will replace speculative traders as the agents of price determination, and in which it becomes the main function of prices to guide production rather than to reflect the merely immediate and temporary conditions of demand and supply.

The main fault with the present price-registering system is its disregard for long-time cost-of-production price. The changes in the price of hogs, for example, instead of bearing some constant relation to cost of production are only the response of the market to the day's receipts of hogs, to the seasonal demand for pork, and to the strategy of the packers. Cost means the industrial requirements of production, expressed in pecuniary terms. Thus, the basic industrial fact about the production of hogs is that it requires approximately 11.5 bushels of corn to make 100 pounds of hog. This is the corn-hog ratio, and "this ratio is cost of production in the very truest sense of the term" (p. 11). The experience since the Civil War has shown that when prices are averaged by decades "hogs have sold on the Chicago market at a price per hundred weight equal to the Chicago price of 11.5 bushels of corn" (p. 28). The

actual price from day to day and from month to month, responding to the surface indications of supply and demand, have fluctuated above and below this "right price" as indicated by the ratio. Supply, demand, and price being mutually interdependent, the adjustment of price merely for the purpose of clearing the day's market results in violent fluctuations in the weekly shipment of hogs, in needless disturbances of the consumer's demand for pork, and in price aberrations inconsistent with the necessary cost of production.

Economic theory has ordinarily assumed that the price (actual) which clears the market is also the price (normal) which will best guide production; the author, however, holds that the two prices are not identical, and he prefers the method of price determination which will keep in mind the second function. He does not minimize the inherent difficulties of dealing separately with the two functions. Once the primary purpose in establishing a price becomes the stabilization of the conditions of production, then the other function of adjusting the current demand to the current supply may need to be performed by some machinery other than price changes. Perhaps a coöperative selling organization would control the flow of the product to market; or cold storage facilities would be used to smooth out the unavoidable irregularities in production and consumption; or even, if necessary in order to stabilize the live-stock industry, perhaps meat products would be sold at a loss for a season to be offset by more profitable prices at a later time. Whatever the substitution, it must at least be evident that prices of corn and of hogs which one year break up herds and bring brood sows to market and which next year hold out false promises to the feeders, no matter how effective they may be as means of equating demand and supply, cannot be regarded as satisfactory guides to production.

Given the present method of price determination, the author believes farmers should organize for collective bargaining. They should reduce their crops at strategic moments, as business concerns curtail output when large production is unprofitable; they should ask, as does organized labor, "At what season of the year is it best to strike?" When once the farmer is as effectively organized as are the other parties to the market controversy, the futility of sabotage may become apparent, and possibly all classes may "unite in production as a source of profit, rather than on clever bargaining" (p. 17).

Such a coöperative arrangement between classes, however, the author believes to be impossible under the present *laissez faire* competitive price system. Owners will not permit their plants to be operated at capacity unless they have some assurance that they will not be penalized by falling prices and pecuniary losses. The author offers the ratio method of price determination as a possible solution. He looks to the economists for statistical inquiries which will measure the failure of present prices to perform their functions and which will serve as a basis for a better method of control. Thus, the author turns from the *laissez faire* economist to the statistical economist, and from the business man to the production engineer.

WALTER W. STEWART

Amherst College

The Financial Policy of Corporations, by Arthur Stone Dewing. New York: The Ronald Press Co. 1920. V vols.

The five volumes of Dewing's *Financial Policy of Corporations* are entitled *Corporate Securities, Promotion, Administration of Income, Expansion*, and *Failure and Reorganization*.

When the reviewer had completed the first of these five volumes, he was convinced

that he was in the midst of the finest study in the field of corporation finance that had ever been made in the United States. In the text there is a good, straightforward statement of the principles and conclusions determined upon, and in the copious footnotes are adequate examples and evidence for the generalizations above. With the exception of the treatment of preferred stocks and of the definition of capital (to be discussed in later paragraphs), there is little to criticise in the first volume. Many parts of the other four volumes, however, are not up to the standard set by the first: the footnotes are less copious, and the text not always so convincing. One feels that the material is often thin, and that it could have been compressed into less than five volumes. However, the five volumes as a whole represent the best study in corporation finance that has been made, and the student can well afford to spend the extra time and money demanded.

Most of the statistics presented are not complicated and call for no comment. In Volume III (p. 93) there is one use of statistical data that is very questionable. In order to prove the general principle that the earnings of companies which manufacture inexpensive consumption goods are more regular than the earnings of companies engaged in the manufacture of expensive production goods, the author compares the earnings of the Diamond Match Company with those of the American Locomotive Company. Obviously, even if these two companies had been equally well managed—which, the reviewer is informed, is open to serious question—the evidence furnished by their earnings is insufficient for statistical proof. The principle is certainly logical, but the meager data should have been used merely as a footnote, with all apologies to the statistician, and with the statement that further evidence would be necessary before the generalization could be statistically proved.

Some of the economic theory expounded in the early chapters of Volume IV (*Expansion*) might have been discarded or substantiated had statistical evidence been sought for or adduced. The Federal Trade Commission in some of its studies (see especially the Canned Salmon Report) has begun to obtain statistical data on the costs of various sized plants and companies; the inclusion of such data would have been, to say the least, illuminating.

In a footnote in Volume III (p. 16), the Bureau of Corporations' Steel Report is referred to as "somewhat biased against the corporation investigated." Although the government's investigators may be anxious to find something to criticise, they undoubtedly obtain facts to which an individual cannot have access. In this connection the reviewer is tempted to recall that in Dewing's *Corporate Promotions and Reorganizations* there is a criticism of the Steel Report because it often fails to give a reference or an authority for its statements; obviously, the agents of the Bureau of Corporations were not constrained to depend upon what some man told them, but had original records as evidence. Furthermore, just as the point of view of the government may be "biased," so Dewing's is often just as biased, but in the other direction. The student, who obtains his information from interviews and published reports, should take what he receives with a grain of salt unless he has access to the books of the man interviewed. Dewing's treatment of trade associations, of blue sky stocks, and his belief in the authenticity of published accounting statements suggest the attitude of the conservative, honest investment banker rather than that of the critical economist.

Probably the most serious criticism of *The Financial Policy of Corporations* is the treatment of industrial preferred stocks. The author seems to have a curious prejudice against these securities: in one place, he suggests that they have not been in existence long enough to warrant a judgment as to their value; in other places, he

implies that they are bad investments. He underestimates the cumulative and retirement features. He gives little or no attention to those important preferred stock provisions which make it impossible to put any liens ahead of the preferred issue. Finally, he thinks that the preferred stockholder is a partner, not a creditor. A study of the bulk of the industrial preferred stocks should immediately disclose that there is little difference between this type of security and the bond, particularly if the bond rests on the general credit of the company and not on a specific mortgage. The preferred dividend is merely a high rate of interest, explained by the risk involved. In one place he practically admits that the sinking fund provision makes an issue of preferred seem very much like a funded obligation of the owners, but he fails to recognize that the distinction between the common stockholder and the preferred stockholder is usually almost as definite as the distinction between the common stockholder and the bondholder. The preferred stockholder's right to vote, contingent upon the non-payment of dividends, is no reason for classifying him with a voting common stockholder, but rather with the mortgage bondholder, who has the right to foreclose if he is not paid his interest.

In Volume III there are some extended discussions of the problems of accounting. Much attention is given to depreciation, which is considered a deduction from gross profits in order to reduce gross to net profits. Depreciation, it seems to the reviewer, ought to be included in costs before any kind of profit (economic or accounting) is reached. Depreciation is the cost of fixed capital goods and would be little different from "Raw Materials" (current capital goods) if the accountant did not arbitrarily use a year as a fiscal basis. An economist, giving as much space to accounting problems as Dewing gives, should have presented a more analytical treatment of the problem of revaluing the capital goods on the balance sheet and to the distinction between economic and accounting profit, particularly because these questions and similar ones have a close connection to the interpretation of financial data. In the first pages of the first volume the difficulties encountered in the use of the words capital and capitalization are expressed. It may not be out of place here to suggest two or three definitions which might have helped to clarify the author's interpretation of accounting data. Capital (the basis of the interest charge) might be defined as *postponed claims on consumption goods expressed in terms of money.* Capital is transferred by its owner, the capitalist, to the entrepreneur, who uses it to purchase capital goods and to pay wages, interest, and rent. Dewing, like most writers, uses capital to mean what is here defined as capital goods, or even worse, fixed capital goods. The definitions set forth in the foregoing sentences help to explain why the accountant insists on the original cost of the assets on the balance sheet rather than revaluations, especially when he is using the balance sheet for a determination of the investment or the capital which is to serve as a basis for the interest charge or for the measurement of return (economic profit and interest). New capital injected or profits earned and left in the business also represent postponed claims and are the only additions that should be made to the capital or investment. Capitalization, it seems to the reviewer, should be restricted to mean the sum of the stocks and bonds, and should be definitely distinguished from capital. If the assets were reappraised or goodwill injected, more stock might be issued or the surplus might be written up, but no more capital would be thereby created.

It would be unjust to end this review without some commendation of Professor Dewing's scholarship and without restating the importance of his contribution to the field of corporation finance. The reviewer felt that it would be more practicable in the space allotted (and more valuable for the critical reader) to call attention to what

seemed to him the faults of this study; its good points are so numerous and will be so apparent to the reader that no comment seemed necessary. KEMPER SIMPSON
Department of Agriculture

Government Control of the Sugar Industry in the United States, by Joshua Bernhardt. New York: The Macmillan Company. 1920. x, 272 pp.

A volume which narrates the history of a war-time policy can serve only a peace-time purpose. It is the privilege of the amateurs who will manage future wars to disregard all experience and to muddle things for themselves, even as we did before them. Mr. Bernhardt probably had no thought of putting into the hands of future belligerents a valuable contribution to the literature of economic organization for war. Even if he did, there is little prospect that his volume will carry across the years to a future sugar control board. But none the less, the book is important because of its contribution to our very imperfect understanding of our industries, and of the technical problem of controlling them in such a way as to make them serve appointed ends. Excluded from the laboratory, denied the use of the experimental method, and unable to determine in advance of the fact the consequences of a definite procedure, we should welcome as a windfall the mass of material about the organization of industry and the effects of various methods of controlling it which the war has given us.

Among such contributions the story of the control of sugar is among the most significant. The problem which it presents is a fair sample of the problems involved in the control of industry to a purpose. The period of time, much briefer than the scholars would have had it, is ample for at least a tentative statement of problems. The account given by Mr. Bernhardt is objective enough to warrant careful attention. Despite its authorization by the United States Sugar Equalization Board, it is a straightforward story, so free from bias as to be almost dull. The 128 pages of text are for the most part a running narrative, littered here and there with documents and statistics. And enough of this crude raw material is left to cover 134 pages of appendix.

The narrative covers in detail the three periods into which the history of the control of sugar during the war is to be divided: (1) The period of "voluntary coöperation of various branches of the sugar trade" under the general oversight of the Sugar Division of the Food Administration; (2) the period of the centralized administration of the distribution of sugar by the United States Sugar Equalization Board; and (3) the period of the disturbed and hesitating "demobilization of sugar control" which followed the armistice. Of these the second has the larger meaning for the student of economic organization and deserves careful study. It represents an attempt to secure an equitable distribution of sugar by unified administrative judgments based upon accurate and comprehensive statistical data. A study of policy during this period involved a consideration of (1) the ends which the Board tried to achieve; (2) the general industrial conditions which set the terms of the problem; (3) the methods that were employed; and (4) the results that were effected.

The aims of the sugar administration, reduced from Mr. Bernhardt's very comprehensive list, were: (1) To effect a proper distribution of sugar between ourselves and our allies; (2) to secure an equitable distribution among different sections of the country and different groups of the population; (3) to keep the price of sugar within the reach of the ordinary consumer; and (4) to maintain the largest possible production of this staple commodity. It is evident that these aims were more or less contradictory, and that the purpose of the Board was to secure the best relative realization of them.

The great difficulty of the problem lay in the unusual conditions prevailing in the sugar industry. The war had closed the sugar fields of Central Europe and had left England and France without their customary supply. The dearth of shipping had placed the supply from Java beyond reach. The Allies, accordingly, turned to the West, particularly to Cuba, where the crop was threatened by an insurrection. More important still, since the supply available in late summer and early autumn was most seriously curtailed, the seasonal distribution of the product was seriously disturbed. And, lest the problem escape complications, the coming of prohibition had increased the demand for sugar just at a time when the supply of labor for sugar production was decreasing and costs were rapidly rising.

The policy of the sugar administration aimed at a quick adjustment of the distribution of sugar to these new conditions through a unified control of the supply. First, a comprehensive statistical survey was made of the production and the probable future demand for sugar. Second, by purchase of the Cuban crop and other devices, the Board obtained a nominal control of the whole supply and could allocate it in terms of the country's needs. Third, an apportionment of the supply between ourselves and our allies was effected. Here the scarcity of shipping, which imposed a limit upon the amount which Europe could take, made the problem easy of solution. Fourth, a single uniform price for the refined domestic product and a series of differential prices to the growers in Cuba, Porto Rico, Hawaii, Louisiana, and Michigan were established. The differentials included such items as carriage, import duty, and refining. The solution of each of these problems involved a weighing of many values and a consideration of a vast amount of information presented in quantitative form. Thus, in fixing the price of sugar the Board aimed, on the one hand, to avoid profiteering, and, on the other, to stimulate production. To hit upon the price which would best protect the consumer without seriously threatening the supply, the amounts likely to be available at all possible costs had to be approximately determined.

The descriptive task essayed by Mr. Bernhardt causes him to refrain from a judgment upon the success of this venture into national control. But the net effect of the narrative upon the reader is to raise certain questions which he finds himself unable either to answer or to dismiss. When conditions are changing rapidly, is administration by a centralized agency to be preferred to adjustment "by the natural workings of the price system"? Which of the two is quick and effective, and which imperfect, vacillating, and tardy? Is not the efficiency of the distribution of a commodity by the "trade" effective just to the extent that conditions remain the same and distribution follows routine paths? Could the distribution be improved and the commodity offered for less if a centralized statistical service which gathered all the facts was maintained? How far would a centralized allocation of the product help matters? The evidence presented by Mr. Bernhardt does not make out a case for a change from the present system; but it does create so strong a presumption in favor of a more centralized control, based upon an adequate statistical service, that it requires further examination.

In this connection the evidence afforded by the failure of the government to purchase the Cuban crop for 1919–20 and the consequences of the failure are not without significance. Had the government purchased the crop, the price would doubtless have advanced and we would have had conclusive evidence of the failure of "administrative control." But the events which followed in the wake of "letting sugar alone" through dearth and high prices, gave abundant evidence that the workings of unregulated industry are not always so perfect as to be automatic.

WALTON H. HAMILTON

Amherst College

Report on the Mortality from Influenza in England and Wales during the Epidemic of 1918–1919. Supplement to the Eighty-first Annual Report of the Registrar General of Births, Deaths and Marriages. London: H. M. Stationery Office. 1920. 119 pp.

The epidemic of influenza caused nearly 200,000 deaths in the civilian and military population of England and Wales during the forty-six week period, June 23, 1918, to May 10, 1919. Among the civilian population alone it was estimated that 184,000 deaths occurred. The rate of mortality for the forty-six week period was practically 6,400 per million of population.

The epidemic appeared in three principal waves or phases. The first wave began in the last eight days of June, 1918, reached a maximum in the week ended July 18, and subsided about the first of September. The second wave or phase began about the third week in September, rose to a maximum in the week ended November 9, and attained the minimum toward the last of the following January. The late winter and spring wave had its onset about February 1, the maximum in the week ended March 1, and the minimum approximately in the week ended May 10. Among the civilian population the death toll of the summer quarter of 1918 was estimated to be 17,500, during the last quarter of 1918, 115,000, and during the period January 1 to May 10, 51,500. Some 14,000 deaths from influenza and its consequences were experienced in the military establishment in England and Wales during the forty-six week period.

The detailed general tables in the report are introduced by a statement of the several methods used in estimating the epidemic damage for the forty-six week period. A tabular review of the age distribution of deaths from influenza in a "standard" female population, for the years 1847 and 1848, and for each year from 1890 to 1918, is then shown. Proportions of influenza deaths at five-year age groups in the calendar quarters of 1918 and in the first quarter of 1919 are given to display the unusual age incidence of the prevailing disease. The type of influenza which occurred during the first twenty-five weeks of 1918 seemed to affect chiefly young children and old people; but toward the end of June an abrupt shift in the age incidence of the disease occurred. A sudden increase in the proportion of deaths at the age group 15 to 24 years was observed; then, as the summer epidemic approached the peak, the age ranges between 25 and 35 years were most affected. The same phenomenon of an increase in the proportion of deaths in adolescence was noted at the outbreak of the autumnal and most violent phase of the epidemic. During the week of maximum death rate, the week ended November 9, the proportion of deaths was highest in the age division 20 to 30 years.

From the evidence submitted in the report it would seem that the northern areas were first affected, and that in the first wave the mortality decreased from north to south. The second wave began approximately in the week ended September 28, about two weeks later than the commencement of the epidemic in the northeastern part of the United States (week ended September 14). The interval between the second and third waves was much shorter than that between the first and second. During the first intermission in the late summer of 1918, the epidemic mortality remained below 2 per 1,000 for eleven weeks, while the intermission was only five weeks between the second and third waves.

An interesting section (pp. 27–30) is devoted to tables of crude data on the possible relation of influenza damage to such factors as wealth, general health conditions, and the prevalence of other infectious diseases. Tables on diseases and conditions complicating influenza are then displayed. An important table (XVIII) is shown on the

proportion of influenza deaths associated with puerperal diseases and conditions. A graphic section (pp. 38–47) illustrates some of the general tables discussed in the introductory section of the report. Table XX (pp. 48–80) records the number of influenza deaths, and death rates per 1,000, for each of the forty-six weeks in the principal civil divisions of England and Wales.

It was of course impossible in the time available for the Registrar General to have had analytic studies prepared. Most of the conclusions regarding the initial areas affected by the several waves of the epidemic, the relative severity in populations of varying density and economic condition, the comparative damage done in each of the waves in the numerous registration districts, and other points of interest, were drawn from inspection of the crude results. It is hoped that students of the pathometry of influenza will avail themselves of the materials which the Registrar General has set before them and endeavor by approved methods of analysis to provide concise descriptions of the data.

The Ministry of Health of Great Britain has issued a comprehensive report on the influenza epidemic of 1918–1919.* Some of the chapters will be of interest to students of influenza pathometry. It is proposed to review these statistical chapters in a later number of the QUARTERLY.

E. W. KOPF

The Ownership and Valuation of Mineral Properties in the United Kingdom, by R. A. S. Redmayne and Gilbert Stone. London: Longmans, Green & Co. 1920. 256 pp.

One lays down Sir Richard Redmayne's latest book with the feeling that they do things better in England. He and his collaborator, Gilbert Stone, have brought together under a single cover the essential facts concerning property rights in minerals, methods of valuation for sale and taxation, and the present system of taxation in England, information which could be found in America, if at all, only after arduous search through many scattered sources. There is no book covering the American field like their *Ownership and Valuation of Mineral Properties in the United Kingdom*. The nearest approach to it consists of chapters on valuation in standard works on mining, such as those of Hoover, Finlay, and Young, and of papers by Chance, Brinsmade, Finlay, Allen, and others, scattered through the technical press and through the Proceedings of the American Institute of Mining and Metallurgical Engineers. The publications of the state and federal governments have dealt for the most part with specialized phases of the subject only, such as the valuation of iron mines or oil and gas wells for purposes of taxation, or the valuation of mineral-bearing public lands for sale or lease.

The authors are eminently qualified for the task they have undertaken. To it Redmayne brings not only ripe experience as colliery manager, consulting engineer, and professor of mining at the University of Birmingham, but also a record of years of public service as Chief Inspector of Mines for the United Kingdom, and as Assistant Fuel Controller during the war. Few men in England are entitled to speak with more authority on the mining industry than he. Mr. Stone's share in the work is apparently the portion relating to the law of mineral properties, and the combination of the viewpoint of the lawyer with the engineer is a happy one.

The book is designed primarily to assist the mining expert. It is, however, so

* *Report on the Pandemic of Influenza, 1918–1919.* London: H. M. Stationary Office. 1920. 577 pp.

clearly written as to be comprehensible to the layman, and it contains much descriptive matter of value to the student of economics. Partly because of the interests of its authors, partly because of the overwhelming importance of coal in the mineral industry of Great Britain, the book deals chiefly with coal properties, and its lessons for Americans are largely confined to coal.

Three chapters are devoted to the rights of owners and tenants, lessors and lessees, in mineral-bearing lands. Because of differences between English and American law with respect to property in land these are of limited interest to American engineers and economists. Of more immediate concern are the chapters on valuation for purposes of taxation and on the present régime of parochial and national taxes. The workings of the mineral rights duty, the excess mineral rights duty, and the increment value duties, as applied to coal lands, and also of the income tax as applied to colliery operations, are described. Taxation of coal-bearing lands is shown to be for the most part on the basis of royalties and rents, *i.e.*, on tonnage produced, not on the value of the coal in the ground. It is significant to note that four years before the war England had adopted a tax on the unearned increment in the value of mineral-bearing lands.

Two chapters on rents and royalties develop the fact that in few cases is the English coal operator at the same time the owner of the coal; instead he leases the coal bed and a wayleave to get access to it. The number of coal owners in the United Kingdom is stated to be about four thousand. Royalties differ from one field to another; a common form is shown to be a minimum fixed rental with provision for a royalty in addition of so much per ton on the coal produced above the amount necessary to make up the rental. The average royalty (pp. 72, 140) is thought by the authors to be five and one-half pence per ton, or translated into American units, about nine and one-half cents per net ton.

In the chapters dealing with valuation primary emphasis is laid on the valuation of holdings in coal lands rather than of producing collieries, though some attention is paid to the latter as well. American engineers will note with interest the constants for depreciation of plant and equipment, given on pages 123–4. Formulas are presented for the determination of present values, and much attention is given by the authors to the proper rate of interest to be assumed (a) to reimburse the investor for the risk of his original capital, and (b) to amortize the original investment by the creation of a sinking fund. The former the authors place at from 8 to 10 per cent on an ordinary mining risk, the latter at from 3 or 4 per cent in pre-war times, and from 5 to 6 per cent at present.

Throughout the book there runs a consciousness of impending changes in the coal-mining industry. As pertinent to the contemporary discussion of nationalization of coal mines the authors present an estimate of the value of the coal royalties in the United Kingdom, *i.e.*, the purchase price of the coal beds as distinct from the surface rights and from the investment in mining plant and equipment. This they place at approximately 70,000,000 pounds sterling, and they note that estimates of other values have ranged as high as 80,000,000 and 90,000,000 pounds sterling. The cost to the British Nation of expropriating the coal deposits of the country, allowing just compensation to the present owners, would thus be about the same as the annual naval estimates, which are reported as 82,000,000 pounds sterling net for the year 1921–22.

It is the assumption of the authors that in the event of nationalization of the coal deposits the state would continue to exact royalties from the operating companies (p. 86). "Only the consumer," they argue, "would gain by any reduction in royal-

ties. . . . The ultimate result would be a reduced selling price and the stoppage of the inferior collieries unable to bring down their working costs to the required point." The authors have apparently fallen into an error shared by certain members of their respective professions in America. In the long run royalties are not an element in the price of coal to the consumer, but like the rent of land, are a differential between the return on marginal coal properties and more profitable ones. The price of coal in the United Kingdom is normally a competitive price, determined largely by the pull of the export demand upon the supply at home. In competing abroad with inferior foreign coals British producers have an advantage which they have been compelled to share with British coal owners in the form of royalties. In the absence of price fixation by the government the remission of royalties would swell the profits of the more favorably situated producers rather than lower the price to consumers.

F. G. Tryon

United States Geological Survey

The Movement of Wages in the Cotton Manufacturing Industry of New England Since 1860, by Stanley E. Howard. Boston: National Council of Cotton Manufacturers. 1920. 99 pp.

This report, published by the National Council of Cotton Manufacturers, again makes use of that increasingly popular device, the United States Bureau of Labor Index of Wholesale Commodity Prices, as a measure of the progressive increase of real as opposed to money wages for long-time comparisons. The extent of the rise of real wages in this case is so startling as to call into question the legitimacy of the use of that index as a measure of money wages in terms of cost of living.

Aside from a very interesting and competent digression into the inadequacies of the Fall River Experiment, which attempted to fix wages on the basis of a sliding scale based on the margin of profit, the report confines itself to the construction of a single index of wages in the Cotton Manufacturing Industry of New England since 1860. This is done to make possible a comparison of that index with the wholesale price index of the Bureau of Labor as a measure of the rise of real wages. The materials are taken from the reports of investigations of wages in the cotton industry made by the Weeks and Aldrich Investigating Commissions and by the United States Bureau of Labor. The construction of this wage index is a very careful piece of work. Supplementary historical and statistical data are included to throw as much light as possible on the causes of wage fluctuations owing to general business conditions, or to conditions peculiar to the industry, both technical and financial. Also, some attempt is made to show the degree and nature of unemployment and the extent to which all crafts within the industry were affected by wage changes, though such conclusions are necessarily rough because of the limitation of material.

The wage index shows that money wages in the industry in Massachusetts stood at 276.9 in 1915, as compared with 207.3 in 1911 before the Lawrence strike, and 100. in 1860. Because of the wide publicity of living conditions in Massachusetts cotton manufacturing centers at the time of the Lawrence strike, that point is as serviceable as any for making concrete comparisons with 1860. In 1911, when money wages stood at 207.3 in Massachusetts as compared with 100. in 1860, 6.61 per cent of the employees were receiving full-time weekly earnings, according to the report, of under five dollars; 50 per cent under eight dollars; and 90 per cent under twelve dollars. This does not take into account the relatively high degree of unemployment. That means that full-time weekly earnings, not counting unemployment, in 1860, would show 50 per cent of the employees receiving less than four dollars, and 90 per cent less

than six dollars. These figures are well grounded as shown by the evidence presented. However, the report goes on to make a comparison of these money earnings with the cost of living, and uses the Bureau of Labor Wholesale Price Index as a measure of the cost of living. This shows that wholesale prices, and therefore presumably the cost of living, stood lower in 1911 than in 1860, 91.6 as compared to 100.

It is this discrepancy that challenges the validity of the use of the wholesale price index as a measure of real wages, especially over such long periods. It raises questions regarding the relation of wholesale to retail prices, and the relation of fluctuations of business prices to prices entering into the cost of living, especially for periods as distant as 1860. To what degree did the cost of living enter into the business system? To what extent were rents reflected in the wholesale price index in 1860? What was the effect on food prices of the less-crowded living conditions, making garden cultivation possible? To what extent was the town fed by the immediate countryside, excluding large-scale speculation in foods, and the transportation and wholesale handling of them?

These and similar questions must be answered before any such conclusions as appear in this report can obtain critical acceptance. Such comparisons are always interesting. Their danger lies in their uncritical employment by those to whom the publishers may distribute this report.

WINFIELD W. RIEFLER

Amherst College

A Study of Women Delinquents in New York State, by Mabel Ruth Fernald, Mary Holmes Stevens Hayes, Almena Dawley, and Beardsley Ruml. New York: The Century Company. 1920. xviii, 542 pp.

This study of women delinquents in New York State, to which Katherine Bement Davis has contributed the preface, is illustrative of the kind of investigation that ought to be carried on in all the prisons of the United States. For centuries we have been trying to reduce the volume of crimes by crude and unscientific methods, making little or no attempt to understand the human beings whom we hanged, tortured, and imprisoned. Why the criminals themselves have been so long overlooked and neglected, and their acts given so much attention by lawmakers will be one of the questions for future historians of civilization to wrangle over.

The book is based on the case histories of women found in certain institutions, together with the histories of a group of women placed on probation by the Night Court of the boroughs of Manhattan and Bronx. Although these institutions represent all the types of institutions in which women are confined in New York State, not enough attention was paid, in the reviewer's opinion, to their location. Some should have been selected that served the needs of purely rural communities in order that all types of delinquent women might be included in the picture. A more serious criticism could be made of the number investigated. Only 587 women were covered by the survey. When this total is divided into various groups, the number in each group is rather small for statistical purposes. One must remember, however, that it is no easy task to obtain data such as has been collected for this group of women, and should be thankful that the number is as large as it is.

While including the data ordinarily presented in statistical studies of delinquents, such as sex, age, marital condition, color, and nature of offense, the investigation deals also with early home conditions, educational background, occupational history, sex irregularities, and mental capacity—matters of far more significance to the seeker

for causes of criminality. The difficulty that is met with in leaving the well-beaten statistical paths lies in the fact that there is no corresponding data for the population of the country as a whole with which to make comparisons. The authors fully recognize this fact and warn the reader constantly against drawing wrong conclusions. One feels, nevertheless, that some very interesting side lights have been thrown on the problem of crime. For instance:

A very large percentage of the women came from poor homes and a still larger percentage from homes in which there was poor parental supervision.

There was a very slight relationship between education and the various aspects of delinquency.

Over half of the women were idle at the time of conviction.

Delinquency and defective intelligence are *not* synonymous.

There was no demonstrable difference between the mental capacity of recidivists and of first offenders.

There was no demonstrable difference in mental capacity between those who use drugs and alcohol and those who do not.

There are indications that environmental factors have more to do with conduct than Goring supposed.

Probably it would have been better to have omitted some of the calculations, as, for example, tables 164 and 223, as the data does not warrant use. Aside from this tendency to push the investigation further than the facts warrant, the work merits high praise for its painstaking quality, and other investigators will find it well worth studying before they try their hand at similar tasks.

LOUIS N. ROBINSON

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